2. Polynomials

Exercise 2.1

1. Question

Find the zeros of each of the following quadratic polynomials and verify the relationship between the zeros and their coefficients:

(i) $f(x) = x^2 - 2x - 8$

(ii) $g(s) = 4s^2 - 4s + 1$

(iii) $h(t) = t^2 - 15$

(iv) $6x^2 - 3 - 7x$

(v) $p(x) = x^2 + 2\sqrt{2x} - 6$

(vi) $q(x) = \sqrt{3}x^2 + 10x + 7\sqrt{3}$

(vii) $f(x) = x^2 - (\sqrt{3} + 1)x + \sqrt{3}$

(viii) $g(x) = a(x^2 + 1) - x(a^2 + 1)$

Answer

(i) $f(x) = x^2 - 2x - 8$ factorize the given polynomial by splitting the middle term:

⇒ $x^2 - 4x + 2x - 8$

⇒ x (x - 4) + 2 (x - 4)

For zeros of f(x), f(x) = 0

⇒(x + 2) (x - 4) = 0x+2=0x=-2x-4=0x=4

⇒x = -2, 4

Therefore zeros of the polynomial are -2 & 4

In a polynomial the relations hold are as follows:sum of zeroes is equal to $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ product of zeroes is equal to $\frac{Constant\ term}{Coefficient\ of\ x^2}$
For the given polynomial,

Sum of zeros = -2 + 4 = 2

And $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ is -(-2) = 2

Hence the value of $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ and sum of zeroes are same.

Product of zeros = -2 × 4 = -8

$\frac{Constant\ term}{Coefficient\ of\ x^2}$ is -8.

Hence the value of $\frac{Constant\ term}{Coefficient\ of\ x^2}$ and product of zeroes are same.

(ii) $g(s) = 4s^2 - 4s + 1$

factorize the given polynomial by splitting the middle term:

⇒ $4s^2 - 2s - 2s + 1$

⇒ 2s (2s - 1) -1 (2s - 1)

For zeros of g(s), g(s) = 0

(2s - 1) (2s - 1) = 0

2s - 1=0 $s = \frac{1}{2}$

s = $\frac{1}{2}, \frac{1}{2}$

Therefore zeros of the polynomial are $\frac{1}{2}, \frac{1}{2}$

In a polynomial the relations hold are as follows: sum of zeroes is equal to $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ product of zeroes is equal to $\frac{Constant\ term}{Coefficient\ of\ x^2}$

For the given polynomial,

Sum of zeros = $\frac{1}{2} + \frac{1}{2} = 1 \frac{-coefficient\ of\ s}{coefficient\ of\ s^2} = \frac{4}{4} = 1$

Hence the value of $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ and sum of zeroes are same.

Product of zeros = $-\frac{1}{2} \times -\frac{1}{2} = \frac{1}{4} \frac{constant\ term}{coefficient\ of\ s^2} = \frac{1}{4}$

Hence the value of $\frac{Constant\ term}{Coefficient\ of\ x^2}$ and product of zeroes are same.

(iii) $h(t) = t^2 - 15$ use the formula $a^2 - b^2 = (a + b)(a - b)$ to solve the above equation, Here a is t and b is $\sqrt{15}$.

Solve the given expression as: $t^2 - (\sqrt{15})^2 = (t + \sqrt{15})(t - \sqrt{15})$

For zeros of h(t), h(t) = 0

$t + \sqrt{15} = 0$

$t = -\sqrt{15}$

$t - \sqrt{15} = 0$

$t = \sqrt{15}$

Therefore zeros of the given polynomial are t = √15 & -√15

In a polynomial the relations hold are as follows:sum of zeroes is equal to $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$product of zeroes is equal to$\frac{Constant\ term}{Coefficient\ of\ x^2}$For the given polynomial,

Sum of zeros = √15 + (- √15) = 0

The value of $-\frac{Coefficient\ of\ t}{Coefficient\ of\ t^2}$ is 0.

Hence, the value of $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$ and sum of zeroes are same.

Product of zeros $= -\sqrt{15} \times \sqrt{15}$

The value of $\frac{Constant\ term}{Coefficient\ of\ t^2}$ is -$\sqrt{15}$.

Hence the value of $\frac{Constant\ term}{Coefficient\ of\ x^2}$ and product of zeroes are same.

(iv) f(x) = $6x^2 - 3 - 7x$

Write the equation in the form of $ax^2 + bx + c$ as:

$6x^2 - 7x - 3$

factorize the given polynomial by splitting the middle term:

⇒ $6x^2 - 9x + 2x - 3$

⇒ 3x(2x - 3) +1(2x - 3)

⇒ (3x + 1) (2x - 3)

For zeros of f(x),f(x) = 0

⇒ (3x + 1) (2x - 3) = 0

x = $\frac{-1}{3}, \frac{3}{2}$

Therefore zeros of the polynomial are $\frac{-1}{3}, \frac{3}{2}$

In a polynomial the relations hold are as follows:sum of zeroes is equal to $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$product of zeroes is equal to$\frac{Constant\ term}{Coefficient\ of\ x^2}$

Sum of zeros = $\frac{-1}{3}+\frac{3}{2}=\frac{-2+9}{6}=\frac{7}{6}=\frac{7}{6}=\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$

Product of zeros = $\frac{-1}{3}\times\frac{3}{2}=\frac{-3}{6}=\frac{-1}{2}=\frac{constant\ term}{coefficient\ of\ x^2}$

(v) $p(x)=x^2+2\sqrt{2x}-6$ P (x) = x^2 + 3√2x - √2x - 6

For zeros of p(x), p(x) = 0

⇒ x (x + 3√2) -√2 (x + 3√2) = 0

⇒ (x - √2) (x + 3√2) = 0

x = √2, -3√2

Therefore zeros of the polynomial are √2 & -3√2

Sum of zeros = √2 -3√2 = -2√2 = -2√2 = $\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$

Product of zeros = √2 × -3√2 = -6 = -6 = $\frac{constant\ term}{coefficient\ of\ x^2}$

(vi) q (x) = √3x^2 + 10x + 7√3

⇒ √3x^2 + 10x + 7√3

⇒ √3x^2 + 7x + 3x + 7√3

⇒ √3x (x +$\frac{7}{\sqrt{3}}$) + 3 (x +$\frac{7}{\sqrt{3}}$)

⇒ (√3x + 3) (x +$\frac{7}{\sqrt{3}}$)

For zeros of Q(x), Q(x) = 0

(√3x + 3) (x +$\frac{7}{\sqrt{3}}$) = 0

X = $\frac{-3}{\sqrt{3}},\frac{-7}{\sqrt{3}}$

Therefore zeros of the polynomial are $\frac{-3}{\sqrt{3}},\frac{-7}{\sqrt{3}}$

Sum of zeros = $\frac{-3}{\sqrt{3}} + \frac{-7}{\sqrt{3}} = \frac{-10}{\sqrt{3}} = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$

Product of zeros = = $\frac{-3}{\sqrt{3}} \times \frac{-7}{\sqrt{3}} = 7 = 7 = \frac{Constant\ term}{Coefficient\ of\ x2}$

(vii) $f(x) = x^2 - (\sqrt{3} + 1)x + \sqrt{3}$

$f(x) = x^2 - \sqrt{3}x - x + \sqrt{3}$

$f(x) = x(x - \sqrt{3}) - 1(x - \sqrt{3})$

$f(x) = (x - 1)(x - \sqrt{3})$

For zeros of f(x), f(x) = 0

$(x - 1)(x - \sqrt{3}) = 0$

$X = 1, \sqrt{3}$

Therefore zeros of the polynomial are 1 & √3

Sum of zeros = $1 + \sqrt{3} = \sqrt{3} + 1 = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$

Product of zeros = $1 \times \sqrt{3} = \sqrt{3} = \frac{constant\ term}{coefficient\ of\ x^2}$

(viii) $g(x) = a(x^2 + 13) - x(a^2 + 1)$

$g(x) = ax^2 - a^2x - x + ag(x) = ax^2 - (a^2 + 1)x + a$

$g(x) = ax(x - a) - 1(x - a)$

$g(x) = (ax - 1)(x - a)$

For zeros of g(x), g(x) = 0

$(ax - 1)(x - a) = 0$

$X = \frac{1}{a}, a$

Therefore zeros of the polynomial are $\frac{1}{a}$ & a

Sum of zeros

$$= \frac{1}{a} + a$$

$$= \frac{1 + a^2}{a}$$

$$= \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$$

Product of zeros = $\frac{1}{a}$ × a = 1 = 1 = $\frac{constant\ term}{coefficient\ of\ x^2}$

2. Question

If α and β are the zeros of the quadratic polynomial $p(x) = 4x^2 - 5x - 1$, find the value of $\alpha^2\beta + \alpha\beta^2$

Answer

α and β are the zeros of the quadratic polynomial $p(x) = 4x^2 - 5x - 1$

Sum of the roots = $\alpha + \beta$ = $\frac{constant\ term}{coefficient\ of\ x^2}$ = $-\frac{(-5)}{4} = \frac{5}{4}$

Product of the roots = $\alpha \times \beta$ = $\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ = $\frac{-1}{4}$

Now,

$\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$

On substituting values from above, we get

$= \frac{-1}{4} \times \frac{5}{4} = -\frac{5}{16}$

3. Question

If α and β are the zeros of the quadratic polynomial $f(t) = t^2 - 4t + 3$, find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.

Answer

Let α and β are the roots of the given eqn

Sum of the roots = $\alpha + \beta = -\frac{Coefficient\ of\ x}{Coefficient\ of\ x2} = -\frac{(-4)}{1} = 4$

Product of the roots = $\alpha \times \beta = \frac{Constant\ term}{Coefficient\ of\ x2} = \frac{3}{1} = 3$Now, to evaluate

$\alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta)$

On substituting values from above, we get

$\Rightarrow 3^3 \times 4 = 108$

4. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 5x + 4$, find the value of

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Answer

Given: α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 5x + 4$

To find: the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Solution:

α and β are the roots of the given eqn.We know,

Sum of the roots = $\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$

$\Rightarrow \alpha + \beta = -\frac{(-5)}{1} = 5$

AndProduct of the root = $\frac{constant\ term}{coefficient\ of\ x^2}$ $= \frac{4}{1} = 4$

$\Rightarrow \alpha \times \beta = \frac{4}{1} = 4$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$,

$$= \frac{\beta + \alpha - 2\alpha^2\beta^2}{\alpha\beta}$$

$$= \frac{\beta + \alpha - 2(\alpha\beta)^2}{\alpha\beta}$$

On substituting values from above, we get $= \frac{5 - 2(4)^2}{4}$

$$= \frac{5 - 2(16)}{4} = -\frac{27}{4}$$

5. Question

If α and β are the zeros of the quadratic polynomial $p(y) = 5y^2 - 7y + 1$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

Answer

Let α and β are the roots of the given eqn

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = -\frac{(-7)}{5} = \frac{7}{5}$

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{1}{5}$

On substituting values from above, we get

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{(\beta+\alpha)}{\alpha\beta} = \frac{(\alpha+\beta)}{\alpha\beta} = \frac{\frac{7}{5}}{\frac{1}{5}} = 7$$

6. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$.

Answer

Let α and β are the roots of the given eqn

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = -\frac{(-1)}{1} = 1$

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{-4}{1} = -4$

$$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{(\beta+\alpha)}{\alpha\beta} - \alpha\beta$$

On substituting values from above, we get $= \frac{-1}{4} + 4$

$$= \frac{-1+16}{4}$$

$$= \frac{15}{4}$$

7. Question

If α and β are the zeros of the quadratic polynomial $f(x) = 6x^2 + x - 2$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

Answer

Let α and β are the roots of the given eqn

Sum of the roots = $\alpha + \beta = -\frac{Coefficient\ of\ x}{Coefficient\ of\ x2} = -\frac{1}{6}$

Product of the roots = $\alpha \times \beta = \frac{Constant\ term}{Coefficient\ of\ x2} = \frac{-2}{6} = \frac{-1}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha 2 + \beta 2)}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

Using $(a + b)^2 = a^2 + b^2 + 2ab$

On substituting values from above, we get

$\Rightarrow \frac{\frac{-1\times-1}{6\times 6} - \left(\frac{2}{3}\right)}{-\frac{1}{3}} = \frac{-25}{12}$

8. Question

If α and β are the zeros of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$, find the value of

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$.

Answer

Let α and β are the roots of the given eqn

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ = -(-6/3) = 2

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{4}{3}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{(\alpha^2+\beta^2)+2(\alpha+\beta)}{\alpha\beta} + 3\alpha\beta = \frac{(\alpha+\beta)^2 - 2\alpha\beta + 2(\alpha+\beta)}{\alpha\beta} + 3\alpha\beta$

Using $(a + b)^2 = a^2 + b^2 + 2ab$

$$\Rightarrow \frac{(2)^2 - 2\times\frac{4}{3} + 2\times 2}{\frac{4}{3}} + \frac{3\times 4}{3}$$

$$\Rightarrow \frac{4 - \frac{8}{3} + 4}{\frac{4}{3}} + 4$$

$$= 8$$

9. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 + x - 2$, find the value of $\frac{1}{\alpha} - \frac{1}{\beta}$.

Answer

Given : α and β are the zeros of the quadratic polynomial $f(x) = x^2 + x - 2$

To find : the value of $\frac{1}{\alpha} - \frac{1}{\beta}$.

Solution : α and β are the roots of the given eq.

Sum of root: $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-1}{1} = -1$

Product of the roots: $\alpha \times \beta = \frac{constant\ term}{coefficient\ of x^2}$

$$= \frac{-2}{1} = -2$$

Now, $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{\beta - \alpha}{\alpha\beta}$

Use $(a + b)^2 = a^2 + b^2 + 2ab$ and $(a - b)^2 = a^2 + b^2 - 2ab$ to find the value of β-α.
$(\beta+\alpha)^2=\beta^2+\alpha^2+2\beta\alpha$add and subtract 2βα on right hand side of the above equation.
$(\beta+\alpha)^2=\beta^2+\alpha^2+2\beta\alpha+2\beta\alpha-2\beta\alpha(\beta+\alpha)^2=(\beta-\alpha)^2+4\beta\alpha$

$$(\beta - \alpha) = \sqrt{(\beta + \alpha)^2 - 4\alpha\beta}$$

$$\frac{(\beta - \alpha)}{\alpha\beta} = \frac{\sqrt{(\beta + \alpha)^2 - 4\alpha\beta}}{4\alpha\beta}$$

$$\Rightarrow \frac{\sqrt{(-1)^2 - 4(-2)}}{-2} = \frac{\sqrt{9}}{-2} = \frac{-3}{2}$$

$$\therefore \frac{1}{\alpha} - \frac{1}{\beta} = \frac{-3}{2}$$

10. Question

If one zero of the quadratic polynomial $f(x) = 4x^2 - 8kx - 9$ is negative of the other, find the value of k.

Answer

Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is $-\alpha$

Sum of the roots = $\alpha - \alpha$ = $\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ = $-\frac{-8k}{4}$ = 2k

$\alpha - \alpha$ = 2k

0 = 2k

therefore,

k = 0

11. Question

If the sum of the zeros of the quadratic polynomial $f(t) = kt^2 + 2t + 3k$ is equal to their product, find the value of k.

Answer

Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

Sum of the roots = $\alpha + \beta$ = $-\frac{Coefficient\ of\ x}{Coefficient\ of\ x2}$ = $-\frac{2}{k}$

Product of the roots = $\alpha \times \beta$ = $\frac{Constant\ term}{Coefficient\ of\ x2}$ = $\frac{3k}{k}$ = 3

According to the question: sum of zeros = product of zeros

$-\frac{2}{k}$ = 3

k = $-\frac{2}{3}$

12. Question

If the squared difference of the zeros of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

Answer

Consider $f(x) = x^2 + px + 45$,Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

Sum of the roots = $\alpha + \beta$ = $\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ = $-\frac{p}{1}$ = -p

Product of the roots = $\alpha \times \beta$ = $\frac{constant\ term}{coefficient\ of\ x^2}$ = 45

According to the question: squared difference of the zeros = 144

$$(\alpha - \beta)^2 = 144$$

Apply the formula $(x-y)^2 = x^2 + y^2 - 2xy$

$$\Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta = 144$$

$$\Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta + 2\alpha\beta - 2\alpha\beta = 144$$

Apply the formula $(x+y)^2 = x^2 + y^2 + 2xy$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ = 144

⇒ $(-p)^2 - 4 \times 45 = 144$

⇒ $p^2 - 4 \times 45 = 144$

⇒ $p^2 - 180 = 144$

⇒ $p^2 = 144 + 180$

⇒ $p^2 = 324$

⇒ $p = \pm 18$

13. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - px + q$,

prove that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

Answer

Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

Sum of the roots = $\alpha + \beta = \dfrac{-coefficient\ of\ x}{coefficient\ of\ x^2} = -\frac{-p}{1} = p$

Product of the roots = $\alpha \times \beta = \dfrac{constant\ term}{coefficient\ of\ x^2} = q$

Therefore, we have

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4 + \beta^4}{(\alpha\beta)^2}$$
$$= \frac{(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$
$$= \frac{((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$

Putting the values from above, we get

$$= \frac{(p^2 - 2q)^2 - 2q^2}{q^2}$$
$$= \frac{(p^4 - 4p^2q + 4q^2) - 2q^2}{q^2}$$
$$= \frac{p^4 - 4p^2q + 2q^2}{q^2}$$
$$= \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$$

Hence, Proved.

14. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - p(x+1) - c$, show that $(\alpha + 1)(\beta + 1) = 1 - c$.

Answer

Given: α and β are the zeros of the quadratic polynomial $f(x) = x^2 - p(x+1) - c$

To show: $(\alpha + 1)(\beta + 1) = 1 - c$ ..... (1)

solution: one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

f(x) = x^2 - p(x+1) - cf(x) = x^2 - px - p - c f(x) = x^2 - px - (p + c)

Sum of the roots is: $\alpha+\beta=-\frac{coefficient\ of\ x}{coefficient\ of\ x^2}$

$$\Rightarrow \alpha+\beta=-\frac{(-p)}{1}=p$$

Product of coefficient is: $\alpha\times\beta=\frac{constant\ term}{coefficient\ of\ x^2}$

$$\alpha\times\beta=\frac{-(p+c)}{1}=-(p+c)$$

Solve LHS of (1) to get,

$\Rightarrow (\alpha+1)(\beta+1) = \alpha\beta+(\alpha+\beta)+1$

On substituting values, we get $(\alpha+1)(\beta+1)$ = -(p+c) + p + 1

$\Rightarrow(\alpha+1)(\beta+1) = -p-c+p+1 \Rightarrow (\alpha+1)(\beta+1) = 1-c$ Hence proved

15. Question

If α and β are the zeros of the quadratic polynomial such that α + β = 24 and α - β = 8, find a quadratic polynomial having α and β as its zeros.

Answer

A quadratic equation when sum and product of its zeros is given by:

f(x) = k{x^2 - (sum of zeros)x + product of the zeros}, where k is a constant

α + β = 24 ....(1)

α - β = 8 ....(2)

Adding 1 and 2 we get,α + β + α - β = 24 + 8⇒ 2α = 32⇒ α = 16Substitute value in 1 to get16 + β = 24⇒ β = 24-16⇒ β = 8

α = 16 and β = 8

f(x) = k{x^2 - (24)x + 16 × 8}

f(x) = k(x^2 - 24x + 128)

If we will put the different values of k, we will find the different quadratic equations.

16. Question

If α and β are the zeros of the quadratic polynomial $f(x)=x^2-1$, find a quadratic polynomial whose zeros are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

Answer

A quadratic equation when sum and product of its zeros is given by:

$f(x) = k\{x^2 - (sum\ of\ the\ zeros)x + product\ of\ the\ zeros\}$, where k is a constantConsider the polynomial $f(x) = x^2 - 1$,

Sum of the roots = $\alpha + \beta$ = $\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ $= -\frac{0}{1} = 0$

Product of the roots = $\alpha \times \beta$ = $\frac{constant\ term}{coefficient\ of\ x^2}$ $= -1$

New equation have zeroes as $\frac{2\alpha}{\beta}$ *and* $\frac{2\beta}{\alpha}$,

⇒ Sum of the zeros of new eq = $\frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{2\alpha^2 + 2\beta^2}{\alpha\beta}$ $= \frac{2(\alpha^2+\beta^2)}{\alpha\beta}$

$$= \frac{2\left(\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta\right)}{\alpha\beta}$$ [Using $(a + b)^2 = a^2 + b^2 + 2ab$]

⇒ $\frac{2(\alpha+\beta)^2 - 2\times 2\alpha\beta}{\alpha\beta} = \frac{2(0)^2 - 4\times -1}{-1}$ = -4

Product of the zeros of new eqn = $\frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha}$ = 4

$f(x) = k\{(x^2 - (-4)x + 4\}$

$f(x) = k(x^2 + 4x + 4)$

17. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 3x - 2$, find a quadratic polynomial whose zeros are $\frac{1}{2\alpha + \beta}$ *and* $\frac{1}{2\beta + \alpha}$.

Answer

$f(x) = k(x^2 - 6x + 11)$

A quadratic equation when sum and product of its zeros is given by:

$f(x) = k\{x^2 - (sum\ of\ the\ zeros)x + product\ of\ the\ zeros\}$, where k is a constant

Sum of the roots = $\alpha + \beta = -\frac{Coefficient\ of\ x}{Coefficient\ of\ x2} = -\frac{-3}{1} = 3$

Product of the roots = $\alpha \times \beta = \frac{Constant\ term}{Coefficient\ of\ x2} = -2$

Sum of the zeros of new eqn = $\frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha} = \frac{2\beta+\alpha+2\alpha+\beta}{(2\alpha+\beta)(2\beta+\alpha)}$

$\Rightarrow \frac{3(\alpha+\beta)}{(2\alpha+\beta)(2\beta+\alpha)}$

$$= \frac{3(\alpha+\beta)}{4\alpha\beta+2\alpha^2+2\beta^2+\alpha\beta}$$

$$= \frac{3(\alpha+\beta)}{5\alpha\beta+2\alpha^2+2\beta^2}$$

$$= \frac{3(\alpha+\beta)}{5\alpha\beta+2(\alpha^2+\beta^2)}$$

Now, $= \frac{3(\alpha+\beta)}{5\alpha\beta+2(\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta)}$

Using $(a + b)^2 = a^2 + b^2 + 2ab$ we get,

$$= \frac{3(\alpha+\beta)}{5\alpha\beta+2[(\alpha+\beta)^2-2\alpha\beta]}$$

$$= \frac{3(\alpha+\beta)}{5\alpha\beta+2(\alpha+\beta)^2-4\alpha\beta}$$

$$= \frac{3(\alpha+\beta)}{\alpha\beta+2(\alpha+\beta)^2}$$

$\Rightarrow \frac{3(\alpha+\beta)}{\alpha\beta+2\{(\alpha+\beta)^2\}} = \frac{3\times3}{-2+2\times32} = \frac{9}{16}$

Product of the zeros of new eqn = $\frac{1}{2\alpha+\beta} \times \frac{1}{2\beta+\alpha} = \frac{1}{(2\alpha+\beta)(2\beta+\alpha)}$

$\Rightarrow \frac{1}{(2\alpha+\beta)(2\beta+\alpha)}$

$$= \frac{2}{4\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta}$$

$$= \frac{2}{5\alpha\beta + 2(\alpha^2 + \beta^2)}$$

$$= \frac{2}{5\alpha\beta + 2(\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta)}$$

$$= \frac{2}{5\alpha\beta + 2[(\alpha + \beta)^2 - 2\alpha\beta]}$$

$$= \frac{2}{5\alpha\beta + 2(\alpha + \beta)^2 - 4\alpha\beta}$$

$$= \frac{2}{\alpha\beta + 2(\alpha + \beta)^2}$$

$$= \frac{1}{-2+2\times 32} = \frac{1}{16}$$

$$f(x) = k\{(x^2 - \frac{9}{16}x + \frac{1}{16}\}$$

$$f(x) = k(x^2 - \frac{9}{16}x + \frac{1}{16})$$

18. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 + px + q$, from a polynomial whose zeros are $(\alpha+\beta)^2 (\alpha-\beta)^2$.

Answer

$f(x) = k\{x^2 - 2(p^2 - 2q)x + p^2(p^2 - 4q)\}$

A quadratic equation when sum and product of its zeros is given by:

$f(x) = k\{x^2 - (sum\ of\ the\ zeros)x + product\ of\ the\ zeros\}$, where k is a constant

Sum of the roots = $\alpha + \beta = -\frac{Coefficient\ of\ x}{Coefficient\ of\ x2} = -\frac{p}{1} = -p$

Product of the roots = $\alpha \times \beta = \frac{Constant\ term}{Coefficient\ of\ x2} = q$

Sum of the zeros of new eqn = $\alpha 2 + \beta 2 + 2\alpha\beta + \alpha 2 + \beta 2 - 2\alpha\beta$ = $2(\alpha^2 + \beta^2)$ = $2\{(\alpha + \beta)^2 - 2\alpha\beta\}$

⇒ 2×$p2$-2×$2q$ = 2($p2$- $2q$) [Using $(a + b)^2 = a^2 + b^2 + 2ab$]

Product of the zeros of new eqn =$(\alpha + \beta)^2(\alpha - \beta)2$ = $(\alpha + \beta)^2\{(\alpha + \beta)^2 - 4\alpha\beta\}$

⇒ $p2(p2 - 4q)$

$f(x) = k\{(x^2 - 2(p^2 - 2q)x + p^2(p^2 - 4q)\}$

19. Question

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 2x + 3$, find a polynomial whose roots are (i) $\alpha + 2, \beta + 2$ (ii) $\frac{\alpha - 1}{\alpha + 1}, \frac{\beta - 1}{\beta + 1}$.

Answer

(i)

$f(x) = k(x^2 - 6x + 11)$

A quadratic equation when sum and product of its zeros is given by:

$f(x) = k\{x^2 - (sum\ of\ the\ zeros)x + product\ of\ the\ zeros\}$, where k is a constant

Sum of the roots = $\alpha + \beta$ = $\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ $= -\frac{-2}{1} = 2$

Product of the roots = $\alpha \times \beta$ = $\frac{constant\ term}{coefficient\ of\ x^2}$ = 3

Sum of the zeros of new eqn = $\alpha + \beta + 4$ = $2 + 4 = 6$

Product of the zeros of new eqn =$(\alpha + 2)(\beta + 2)$ =$\alpha\beta + 2(\alpha + \beta) + 4 = 3 + 2\times 2 + 4 = 11$

$f(x) = k(x^2 - 6x + 11)$

(ii) A quadratic equation when sum and product of its zeros is given by:

$f(x) = k\{x^2 - (sum\ of\ the\ zeros)x + product\ of\ the\ zeros\}$, where k is a constant

Sum of the roots = $\alpha + \beta$ = $\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ $= -\frac{-2}{1} = 2$

Product of the roots = $\alpha \times \beta$ = $\frac{constant\ term}{coefficient\ of\ x^2}$ = 3

Sum of the zeros of new eqn = $\frac{\alpha-1}{\alpha+1}+\frac{\beta-1}{\beta+1}=\frac{(\alpha-1)(\beta+1)+(\alpha+1)(\beta-1)}{(\alpha+1)(\beta+1)}=$ $\frac{\alpha\beta+\alpha-\beta-1+\alpha\beta+\beta-\alpha-1}{\alpha\beta+(\alpha+\beta)+1}$ $\frac{2\alpha\beta-2}{\alpha\beta+(\alpha+\beta)+1}=\frac{2\times3-2}{3+2+1}=\frac{2}{3}$

Product of the zeros of new eqn $=\frac{\alpha-1}{\alpha+1}\times\frac{\beta-1}{\beta+1}=\frac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)}=\frac{\alpha\beta-(\beta+\alpha)+1}{\alpha\beta+(\alpha+\beta)+1}=\frac{3-2+1}{3+2+1}=\frac{1}{3}$

Therefore eqn is: $f(x)=k\left\{x^2-\frac{2}{3x}+\frac{1}{3}\right\}$

$$\frac{\alpha-1}{\alpha+1}+\frac{\beta-1}{\beta+1}=\frac{\beta-1+\alpha-1}{(\alpha+1)(\beta+1)}=\frac{\alpha+\beta-2}{\alpha\beta+(\alpha+\beta)+1}=\frac{2-2}{\alpha\beta+(\alpha+\beta)+1}=0$$

20. Question

If α and β are the zeros of the quadratic polynomial $f(x)=ax^2+bx+c$, then evaluate:

(i) $\alpha-\beta$ (ii) $\frac{1}{\alpha}-\frac{1}{\beta}$

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$ (iv) $\alpha^2\beta+\alpha\beta^2$

(v) $\alpha^4+\beta^4$ (vi) $\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}$

(vii) $\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}$ (viii) $a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$

Answer

(i) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

$f(x)=ax2+bx+c$

Sum of the roots = $\alpha+\beta=\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}=\frac{-b}{a}$

Product of the roots = $\alpha\times\beta=\frac{constant\ term}{coefficient\ of\ x^2}=\frac{c}{a}$

⇒ On substituting values, we get

$\Rightarrow(\alpha-\beta)=\sqrt{\{(\alpha+\beta)^2-4\alpha\beta\}}=\sqrt{\left\{\left(\frac{-b}{a}\right)^2-4\frac{c}{a}\right\}}=\frac{\sqrt{b^2-4ac}}{a}$

(ii) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

f(x) = ax^2 + bx + c

Sum of the roots = $\alpha + \beta = \dfrac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \dfrac{-b}{a}$

Product of the roots = $\alpha \times \beta = \dfrac{constant\ term}{coefficient\ of\ x^2} = \dfrac{c}{a}$

⇒ On substituting values, we get

$$\Rightarrow \frac{1}{\alpha} - \frac{1}{\beta} = \frac{(\beta-\alpha)}{\alpha\beta} = -\frac{\sqrt{\{(\alpha+\beta)^2 - 4\alpha\beta\}}}{\alpha\beta} = -\frac{\sqrt{\left\{\left(\frac{-b}{a}\right)^2 - 4\frac{c}{a}\right\}}}{\frac{c}{a}}$$

$$= \frac{\sqrt{b^2 - 4ac}}{c}$$

(iii) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

f(x) = $ax^2 + bx + c$

Sum of the roots = $\alpha + \beta = \dfrac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \dfrac{-b}{a}$

Product of the roots = $\alpha \times \beta = \dfrac{constant\ term}{coefficient\ of\ x^2} = \dfrac{c}{a}$

⇒ On substituting values, we get

$$\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{(\beta+\alpha) - 2(\alpha\beta)2}{\alpha\beta} = \frac{(-b/a) - 2(c/a)2}{c/a} = -\left(\frac{b}{c} + \frac{2c}{a}\right)$$

(iv) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

f(x) = $ax^2 + bx + c$

Sum of the roots = $\alpha + \beta = \dfrac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \dfrac{-b}{a}$

Product of the roots = $\alpha \times \beta = \dfrac{constant\ term}{coefficient\ of\ x^2} = \dfrac{c}{a}$

⇒ On substituting values, we get

$\Rightarrow \alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta) = \frac{c}{a} \times \frac{-b}{a} = \frac{-bc}{a2}$

(v) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

f(x) = $ax^2 + bx + c$

Sum of the roots = $\alpha + \beta$ = $\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ = $\frac{-b}{a}$

Product of the roots = $\alpha \times \beta$ = $\frac{constant\ term}{coefficient\ of\ x^2}$ = $\frac{c}{a}$

$\Rightarrow \alpha^4 + \beta^4 = (\alpha^2+\beta^2)^2 - 2(\alpha\beta)2 = \{(\alpha+\beta)^2 - 2\alpha\beta\}2 - 2(\alpha\beta)2$ [Using $(a + b)^2 = a^2 + b^2 + 2ab$]

On substituting values, we get

$\Rightarrow \alpha^4 + \beta^4 = \left\{\left(-\frac{b}{a}\right)^2 - \left(\frac{2c}{a}\right)\right\}^2 - 2\left(\frac{c}{a}\right)^2$

$\Rightarrow \alpha^4 + \beta^4 = \left(\frac{b2-2ac}{a2}\right)2 - 2\left(\frac{c^2}{a^2}\right)$

$\Rightarrow \alpha^4 + \beta^4 = \frac{(b^2-2ac)^2 - 2a^2c^2}{a^4}$

(vi) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

f(x) = $ax^2 + bx + c$

Sum of the roots = $\alpha + \beta$ = $\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$ = $\frac{-b}{a}$

Product of the roots = $\alpha \times \beta$ = $\frac{constant\ term}{coefficient\ of\ x^2}$ = $\frac{c}{a}$

$\frac{1}{a\alpha+b} + \frac{1}{a\beta+b} = \frac{a\beta+b+a\alpha+b}{\{(a\alpha+b)(a\beta+b)\}} = \frac{a(\alpha+\beta)+2b}{\{(a\alpha+b)(a\beta+b)\}}$

⇒ On substituting values, we get

$\frac{a(\alpha+\beta)+2b}{\{(a\alpha+b)(a\beta+b)\}} = \frac{a(-b/a)+2b}{a2\alpha\beta+ab(\alpha+\beta)+b2}$

$\Rightarrow \frac{a(-b/a)+2b}{(\frac{a^2c}{a})+ab(-b/a)+b2} = \frac{b}{ac}$

(vii) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

f(x) = ax^2 + bx + c

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{\beta(a\beta+b)+\alpha(a\alpha+b)}{\{(a\alpha+b)(a\beta+b)\}}$

$\Rightarrow \frac{a(\alpha^2+\beta^2)+b(\alpha+\beta)}{a2\alpha\beta+ab(\alpha+\beta)+b2} = \frac{a(\alpha+\beta)2-2\alpha\beta+b(\alpha+\beta)}{a2\alpha\beta+ab(\alpha+\beta)+b2} = \frac{a(-b/a)2+b\left(-\frac{b}{a}\right)}{(\frac{a^2c}{a})+ab(-b/a)+b2}$

$\Rightarrow \frac{\frac{b}{a}-(\frac{b^2}{a})}{\frac{a^2c}{a}-b2+b2} = -\frac{2}{a}$

(viii) Let one root of the given quadratic polynomial is α

Other root of the given quadratic polynomial is β

f(x) = ax^2 + bx + c

Sum of the roots = $\alpha + \beta = \frac{-coefficient\ of\ x}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the roots = $\alpha \times \beta = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=$

$\Rightarrow \frac{a(\alpha^3+\beta^3)}{\alpha\beta}+\frac{b(\alpha^2+\beta2)}{\alpha\beta} = \frac{\{a(\alpha^3+\beta^3)+b(\alpha^2+\beta^2)\}}{\alpha\beta} = \frac{\{a(\alpha+\beta)3-3\alpha\beta(\alpha+\beta)\}+\{b(\alpha+\beta)^2-2\alpha\beta\}}{\alpha\beta}$

On substituting values, we get

$= \frac{\{a(-b/a)3-3c/a(-b/a)\}+\{b\left(-\frac{b}{a}\right)^2-2c/a\}}{c/a}$

$= \frac{\left\{-\frac{b^3}{a^2}+3bc/a2\right\}-b3/a2-2c/a\}}{c/a}$ = b

Exercise 2.2

1. Question

Verify that the numbers given along side of the cubic polynomials below are their zeros. Also, verify the relationship between the zeros and coefficients in each case:

(i) $f(x) = 2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(ii) $g(x) = x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Answer

(i) $f(x) = 2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$ $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2 =$ $\frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 = \frac{1+1-10+8}{4} = 0$

$f(1) = 2\times13 + 12 - 5\times1 + 2 = 2 + 1 - 5 + 2 = 0$

$f(-2) = 2\times(-2)^3 + (-2)^2 - 5\times(-2) + 2 = -16 + 4 + 10 + 2 = 0$

Let $\alpha = \frac{1}{2}$; β=1; γ=-2

$\alpha + \beta + \gamma = \frac{1}{2} + 1 - 2 = \frac{-1}{2} = -\frac{Coefficient\ of\ x2}{Coefficient\ of\ x3}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2}\times1 + 1\times-2 - 2\times\frac{1}{2} = \frac{-5}{2} = \frac{Coefficient\ of\ x}{Coefficient\ of\ x3}$

$\alpha\beta\gamma = \frac{1}{2}\times1\times-2 = -1 = -\frac{Constant\ term}{Coefficient\ of\ x3}$

(ii) $g(x) = x^3 - 4x^2 + 5x - 2; 2, 1, 1$ $f(2) = 23 - 4(2)^2 + 5\times2 - 2 = 8 - 16 + 10 - 2 = 0$

$f(1) = 13 - 4(1)^2 + 5\times1 - 2 = 1 - 4 + 5 - 2 = 0$

$f(1) = 13 - 4(1)^2 + 5\times1 - 2 = 1 - 4 + 5 - 2 = 0$

Let α =2; β=1; γ=1

$\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = -\frac{Coefficient\ of\ x2}{Coefficient\ of\ x3}$

$\alpha\beta + \beta\gamma + \gamma\alpha = 2\times1 + 1\times1 + 1\times2 = 5 = \frac{Coefficient\ of\ x}{Coefficient\ of\ x3}$

$\alpha\beta\gamma = 2\times1\times1 = 2 = -\frac{Constant\ term}{Coefficient\ of\ x3}$

2. Question

Find a cubic polynomial with the sum, sum of the product of its zeros taken two at a time, and product of its zeros as 3, -1 and -3 respectively.

Answer

A quadratic equation when sum and product of its zeros is given by:

$$f(x) = k\{x^3 - (sum\ of\ the\ zeros)x^2 + (sumof\ zeros\ taken\ two\ at\ a\ time)x^2 - (product\ of\ the\ zeros)\}$$

where k is any non-zero real number.

Here,Sum of zeroes = 3sum of the product of its zeros taken two at a time =-1 product of its zeros = -3

$\Rightarrow f(x) = k\{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$

$\Rightarrow$ f$(x) = k\{x^3 - 3x^2 - x + 3\}$, where k is any non-zero real number.

3. Question

If the zeros of the polynomial $f(x) = 2x^3 - 15x^2 + 37x - 30$ are in A.P., find them.

Answer

Let $\alpha = a - d, \beta = a\ and\ \gamma = a + d$ are the zeros of the given polynomial.

Sum of the zeros = $-\frac{Coefficient\ of\ x2}{Coefficient\ of\ x3}$

$\Rightarrow \alpha + \beta + \gamma = a - d + a + a + d = -\frac{-15}{2}$

$\Rightarrow 3a = \frac{15}{2} = \frac{5}{2}$

$\Rightarrow a = \frac{5}{2}$

$\alpha\beta\gamma = (a - d)\ a(\ a + d) = -\frac{Constant\ term}{Coefficient\ of\ x3} = \frac{30}{2} = 15$

$(a - d)\ a(\ a + d) = 15$

$a(\ a^2 - d2) = 15$

On substituting $a = \frac{5}{2}$

$\left(\frac{5}{2}\right)\left\{\left(\frac{5}{2}\right)2(-d^2)\right\} = 15$

$d2 = 6 - \frac{25}{4} = \frac{1}{4}$

$d = \pm\frac{1}{2}$

$\alpha = a - d = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

$\gamma = a + d = \frac{5}{2} + \frac{1}{2} = \frac{6}{2} = 3$

$Therefore\ zeros\ of\ the\ given\ equation\ are:\ 2, 3, \frac{5}{2}$

4. Question

Find the condition that the zeros of the polynomial $f(x) = x^3 + 3px^2 + 3qx + r$ may be in A.P.

Answer

Let $\alpha = a - d, \beta = a\ and\ \gamma = a + d$ are the zeros of the given polynomial.

Sum of the zeros = $\frac{-coefficient\ of\ x^3}{coefficient\ of\ x^2}$

$\Rightarrow \alpha + \beta + \gamma = a - d + a + a + d = -3p$

$\Rightarrow 3a = -3p$

$\Rightarrow a = -p$

Since a is the zero of the polynomial, therefore $f(a) = 0$

$\Rightarrow f(a) = a^3 + 3pa^2 + 3qa + r = 0$

$\Rightarrow a^3 + 3pa^2 + 3qa + r = 0$

$On\ substituting\ a = -p$, we get

$(-p)^3 + 3p(-p)^2 - 3pq + r = 0$

$\Rightarrow -p^3 + 3p^3 - 3pq + r = 0 \Rightarrow 2p^3 - 3pq + r = 0$

5. Question

If the zeros of the polynomial $f(x) = ax^3 + 3bx^2 + 3cx + d$ are in A.P., prove that $2b^3 - 3abc + a^2d = 0$.

Answer

Let $\alpha = A - D, \beta = A\ and\ \gamma = A + D$ are the zeros of the given polynomial.

Sum of the zeros = $-\frac{Coefficient\ of\ x2}{Coefficient\ of\ x3}$

$\Rightarrow \alpha + \beta + \gamma = A - D + A + A + D = -\frac{3b}{a}$

$\Rightarrow 3A = -\frac{3b}{a}$

$\Rightarrow A = -\frac{b}{a}$

Since A is the zero of the polynomial, therefore $f(A) = 0$

$\Rightarrow f(A) = aA3 + 3bA2 + 3cA + d = 0$

On substituting $A = -\frac{b}{a}$, we get

$$\Rightarrow a(-\frac{b}{a})3 + 3b(-\frac{b}{a})2 + 3c(-\frac{b}{a}) + d = 0$$

$$\Rightarrow -\frac{b^3}{a^2} + \frac{3b^3}{a^2} + \frac{3bc}{a} + d = 0$$

$$\Rightarrow \frac{-b3 + 3b3 + 3abc + a2d}{a^2} = 0$$

$\Rightarrow -b^3 + 3b^3 + 3abc + a^2d = 0$

$\Rightarrow 2b^3 + 3abc + a^2d = 0$

6. Question

If the zeros of the polynomial $f(x) = x^3 - 12x^2 + 39x + k$ are in A.P., find the value of k.

Answer

Let $\alpha = a - d, \beta = a \text{ and } \gamma = a + d$ are the zeros of the given polynomial.

Sum of the zeros = $-\frac{Coefficient\ of\ x2}{Coefficient\ of\ x3}$

$\Rightarrow \alpha + \beta + \gamma = a - d + a + a + d = 12$

$\Rightarrow 3a = 12$

$\Rightarrow a = 4$

Since a is the zero of the polynomial, therefore $f(a) = 0$

$\Rightarrow f(a) = a3 - 12a2 + 39a + k = 0$

On substituting $a = 4$

$\Rightarrow f(a) = 43 - 12\times42 + 39\times4 + k = 0$

$\Rightarrow 64 - 192 + 156 + k = 0$

$\Rightarrow k = -220 + 192$

$\Rightarrow k = -28$

Exercise 2.3

1. Question

Apply division algorithm to find the quotient q(x) and remainder r(x) on dividing f(x) by g(x) in each of the following:

(i) $f(x) = x^3 - 6x^2 + 11x - 6,\ g(x) = x^2 + x + 1$

(ii) $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3,\ \ g(x) = 2x^2 + 7x + 1$

(iii) $f(x) = 4x^3 + 8x + 8x^2 + 7,\ g(x) = 2x^2 - x + 1$

(iv) $f(x) = 15x^3 - 20x^2 + 13x - 12,\ \ g(x) = 2 - 2x + x^2$

Answer

(i) $f(x) = x3 - 6x2 + 11x - 6$ and $g(x) = x2 + x + 1$

Degree of $f(x)\ is\ 3\ and\ degree\ of\ g(x)\ is\ 2$; therefore degree of $q(x)\ is\ 3 - 2 = 1$ and degree of remainder is less than 2,

Let $q(x) = ax + b$ and $r(x) = cx + d$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$f(x) = q(x) \times g(x) + r(x)$

On substituting values in the above relation we get,

$x3 - 6x2 + 11x - 6 = (ax + b) \times (x^2 + x + 1) + \ (cx + d)$

$x3 - 6x2 + 11x - 6 = ax3 + ax2 + ax + bx2 + bx + b + \ cx + d$

$x3 - 6x2 + 11x - 6 = ax3 + x^2(a + b) + x(a + b + c) + (b + d)$

On comparing coefficients we get,

$a = 1$

$a + b = \ -6$

$a + b + c = 11$

$b + d = \ -6$

On solving above equations we get,

$a = 1,\ b = -7,\ c = 17,\ d = 1$

On substituting these values for $q(x)\ and\ r(x)$

$q(x) = ax + b = x - 7$

$r(x) = cx + d = 17x + 1$

(ii) $f(x) = 10x^4 + 17x3 - 62x2 + 30x - 3$ and $g(x) = 2x2 + 7x + 1$

Degree of $f(x)\ is\ 4\ and\ degree\ of\ g(x)\ is\ 2$; therefore degree of $q(x)\ is\ 4 - 2 = 2$ and degree of remainder is less than 2.

Let $q(x) = ax2 + bx + c$ and $r(x) = px + q$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$f(x) = q(x) \times g(x) + r(x)$

On substituting values in the above relation we get,

$$10x^4 + 17x3 - 62x2 + 30x - 3 = (ax2 + bx + c) \times (2x2 + 7x + 1) + (px + q)$$

$$10x^4 + 17x3 - 62x2 + 30x - 3 = 2ax^4 + 7ax3 + ax2 + 2bx3 + 7bx2 + bx + 2cx2 + 7cx + c + px + q$$

$$10x^4 + 17x3 - 62x2 + 30x - 3 = 2ax^4 + (7a + 2b)x3 + (a + 7b + 2c)x2 + (b + 7c + p)x + (c + q)$$

On comparing coefficients we get,

$a = 5$

$7a + 2b = 17$

$a + 7b + 2c = -62$

$b + 7c + p = 30$

$c + q = -3$

On solving above equations we get,

$a = 1, b = -9, c = -2, p = 53; q = -1$

On substituting these values for $q(x)$ *and* $r(x)$

$q(x) = ax2 + bx + c = 5x2 - 9x - 2$

$r(x) = px + q = 53x - 1$

(iii) $f(x) = 4x3 + 8x2 + 8x + 7$ and $g(x) = 2x2 - x + 1$

Degree of $f(x)$ *is* 3 *and degree of* $g(x)$ *is* 2; therefore degree of $q(x)$ *is* $3 - 2 = 1$ and degree of remainder is less than 2,

Let $q(x) = ax + b$ and $r(x) = cx + d$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$f(x) = q(x) \times g(x) + r(x)$

On substituting values in the above relation we get,

$4x3 + 8x2 + 8x + 7 \; = (ax + b) \times (2x2 - x + 1) + \; (cx + d)$

$4x3 + 8x2 + 8x + 7 \; = 2ax3 - ax2 + ax + 2bx2 - bx + b + \; cx + d$

$4x3 + 8x2 + 8x + 7 \; = 2ax3 + x^2(-a + 2b) + x(a - b + c) + (b + d)$

On comparing coefficients we get,

$a = 2$

$-a + 2b = \; 8$

$a - b + c = 8$

$b + d = \; 7$

On solving above equations we get,

$a = 2, b = 5, c = 11, d = 2$

On substituting these values for $q(x)$ *and* $r(x)$

$q(x) = ax + b = 2x + 5$

$r(x) = cx + d = 11x + 2$

(iv) $f(x) = 15x3 - 20x2 + 13x - 12$ and $g(x) = x2 - 2x + 2$

Degree of $f(x)$ *is* 3 *and degree of* $g(x)$ *is* 2; therefore degree of $q(x)$ *is* $3 - 2 = 1$ and degree of remainder is less than 2,

Let $q(x) = ax + b$ and $r(x) = cx + d$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$f(x) = q(x) \times g(x) + r(x)$

On substituting values in the above relation we get,

$15x3 - 20x2 + 13x - 12 \; = (ax + b) \times (x2 - 2x + 2) + \; (cx + d)$

$15x3 - 20x2 + 13x - 12 \; = ax3 - 2ax2 + 2ax + bx2 - 2bx + 2b + \; cx + d$

$15x3 - 20x2 + 13x - 12 \; = ax3 + x^2(-2a + b) + x(2a - 2b + c) + (2b + d)$

On comparing coefficients we get,

$a = 15$

$-2a + b = \; -20$

$2a - 2b + c = 13$

$2b + d = -12$

On solving above equations we get,

$a = 2, b = 10, c = 3, d = -32$

On substituting these values for $q(x)\ and\ r(x)$

$q(x) = x + b = 15x + 10$

$r(x) = cx + d = 3x - 32$

2. Question

Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm:

(i) $g(t) = t^2 - 3, f(t) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $g(x) = x^3 - 3x + 1, f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

(iii) $g(x) = 2x^2 - x + 3, f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

Answer

(i) $g(t) = t2 - 3$ and $f(t) = 2t^4 + 3t^3 - 2t2 - 9t - 12 = 0$

Degree of $f(t)\ is\ 4\ and\ degree\ of\ g(t)\ is\ 2$; therefore degree of $q(t)\ is\ 4 - 2 = 2$ and degree of remainder is of degree 1 or less,

Let $q(t) = at2 + bt + c$ and $r(t) = pt + q$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$f(t) = q(t) \times g(t) + r(t)$

On substituting values in the above relation we get,

$2t^4 + 3t^3 - 2t2 - 9t - 12 = (at^2 + bt + c)(t^2 - 3) + pt + q$

$2t^4 + 3t^3 - 2t2 - 9t - 12 = at^4 - 3at2 + bt3 - 3bt + ct2 - 3c + pt + q$

$2t^4 + 3t^3 - 2t2 - 9t - 12 = at^4 + bt3 - 3at2 + ct2 - 3bt + pt - 3c + q$

$2t^4 + 3t^3 - 2t2 - 9t - 12 = at^4 + bt3 - t^2(3a + c) - t(3b + p) - 3c + q$

On comparing coefficients we get,

$a = 2$

$b = 3$

$3a + c = 2$

$3b + p = 9$

$-3c + q = -12$

On solving above equations we get,

$a = 2, b = 3, c = -4, p = 0, q = 0$

On substituting these values for $q(t)\ and\ r(t)$

$q(t) = at2 + bt + c\ = 2t2 + 3t - 4$

$r(t) = pt + q\ = 0t + 0 = 0$ Since remainder is zero, therefore $g(t)\ is\ a\ factor\ of\ f(t)$

(ii) $g(x) = x3 - 3x + 1$ and $f(x) = x^5 - 4x3 + x2 + 3x + 1 = 0$

Degree of $f(x)\ is\ 5\ and\ degree\ of\ g(x)\ is\ 3;$ therefore degree of $q(x)\ is\ 5 - 3 = 2$ and degree of remainder is of degree 1 or less,

Let $q(x) = ax2 + bx + c$ and $r(x) = px + q$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$f(x) = q(x)\times g(x) + r(x)$

On substituting values in the above relation we get,

$x^5 - 4x3 + x2 + 3x + 1 = (ax2 + bx + c)(x3 - 3x + 1) + px + q$

$$x^5 - 4x3 + x2 + 3x + 1 = ax^5 - 3ax^3 + ax^2 + bx^4 - 3bx2 + bx + cx3 - 3cx + c + px + q$$

$$x^5 - 4x3 + x2 + 3x + 1 = ax^5 + bx^4 + x^3(c - 3a) + x^2(a - 3b) + x(b - 3c + p) + q + c$$

On comparing coefficients we get,

$a = 1$

$b = 0$

$c - 3a = -4$

$a - 3b = 1$

$b - 3c + p = 3$

$q + c = 1$

On solving above equations we get,

$a = 1, b = 0, c = -1, p = 0, q = 2$

On substituting these values for $q(x)\ and\ r(x)$

$q(t) = ax2 + bx + c \;\; = x2 - 1$

$r(t) = px + q \;\; = 0x + 2 = 2$ Since remainder is 2, therefore $g(x)\, is\, not\, a\, factor\, of\, f(x)$

(iii) $g(x) = 2x2 - x + 3$ and $f(x) = 6x^5 - x^4 + 4x3 - 5x2 - x - 15 = 0$

Degree of $f(x)\, is\, 5\, and\, degree\, of\, g(x)\, is\, 2$; therefore degree of $q(x)\, is\, 5 - 2 = 3$ and degree of remainder is of degree 2 or less,

Let $q(x) = ax3 + bx2 + cx + d$ and $r(x) = px + q$

By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

$f(x) = q(x)\times g(x) + r(x)$

On substituting values in the above relation we get,

$$6x^5 - x^4 + 4x3 - 5x2 - x - 15 = (ax3 + bx2 + cx + d)(2x2 - x + 3\,) + px + q$$

$$6x^5 - x^4 + 4x^3 - 5x^2 - x - 15 = 2ax^5 - ax^4 + 3ax3 + 2bx^4 - bx3 + 3bx2 + 2cx3 - cx2 + 3cx + 2dx2 - dx + 3d + px + q$$

$$6x^5 - x^4 + 4x^3 - 5x^2 - x - 15 = 2ax^5 + (-a + 2b)x^4 + (3a - b + 2c)x^3 + (3b - c + 2d)x^2 + (3c - d + p)x + 3d + q$$

On comparing coefficients we get,

$a = 3$

$-a + 2b = -1$

$3a - b + 2c = 4$

$3b - c + 2d + p = \; -5$

$3c - d + q = \; -1$

$3d + t = \; -15$

On solving above equations we get,

$a = 3,\, b = 1,\, c = -2,\, d = 5\; p = 10,\, q = -30$

On substituting these values for $q(x)\, and\, r(x)$

$q(x) = ax3 + bx2 + cx + d = 3x3 + x2 - 2x + 5$

$r(x) = px + q \;\; = 10x - 30$ Since remainder is $10x - 30$, therefore $g(x)\, is\, not\, a\, factor\, of\, f(x)$

3. Question

Obtain all zeros of the polynomial $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$, if two of its zeros are -2 and -1.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha\ is\ a\ factor\ of\ f(x)$.

Since -2 and -1 are zeros of $f(x)$. Therefore $(x + 2)(x + 1) =\ x2 + 3x + 2$ is a factor of $f(x)$.

Now on dividing $f(x) = 2x^4 + x3 - 14x2 - 19x - 6\ by\ g(x) =\ x2 + 3x + 2$ to find other zeros.

$$x^2 + 3x + 2 \,\big)\, 2x^4 + x^3 - 14x^2 - 19x - 6 \,\big(\, 2x^2 - 5x - 3$$
$$2x^4 + 6x^3 + 4x^2$$
$$- \quad - \quad -$$
$$-5x^3 - 18x^2 - 19x - 6$$
$$-5x^3 - 15x^2 - 10x$$
$$+ \quad + \quad +$$
$$-3x^2 - 9x - 6$$
$$-3x^2 - 9x - 6$$
$$+ \quad + \quad +$$
$$0$$

By applying division algorithm, we have:

$2x^4 + x3 - 14x2 - 19x - 6$ = $(x2 + 3x + 2)(\ 2x2 - 5x - 3)$

$2x^4 + x3 - 14x2 - 19x - 6$ = $(x + 2)(x + 1)(2x2 - 6x + x - 3)$

$2x^4 + x3 - 14x2 - 19x - 6$ = $(x + 2)(x + 1)\{(2x + 1)(x - 3)\}$

Hence, the zeros of the given polynomial are: $-\frac{1}{2}, 3, -2, -1$

4. Question

Obtain all zeros of $f(x) = x^3 + 13x^2 + 32x + 20$, if one of its zeros is -2.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha\ is\ a\ factor\ of\ f(x)$.

Since -2 is zero of $f(x)$. Therefore $(x + 2)$ is a factor of $f(x)$.

Now on divide $f(x) = x^3 + 13x^2 + 32x + 20$, by $(x + 2)$ to find other zeros.

$$x + 2 \,\big)\, x^3 + 13x^2 + 32x + 20 \,\big(\, x^2 + 11x + 10$$
$$x^3 + 2x^2$$
$$- \quad -$$
$$11x^2 + 32x + 20$$
$$11x^2 + 22x$$
$$- \quad -$$
$$10x + 20$$
$$10x + 20$$
$$- \quad -$$
$$0$$

By applying division algorithm, we have:

$x^3 + 13x^2 + 32x + 20 = (x+2)(x^2+11x+10)$We do factorisation here by splitting the middle term,

$\Rightarrow x^3 + 13x^2 + 32x + 20 = (x+2)(x^2+11x+10) \Rightarrow x^3 + 13x^2 + 32x + 20 = (x+2)(x^2+10x+x+10)$

$\Rightarrow x^3 + 13x^2 + 32x + 20 = (x+2)\ \{x(x+10)+1(x+10)\} \Rightarrow x^3 + 13x^2 + 32x + 20 = (x+2)\ (x+10)(x+1)$

Hence, the zeros of the given polynomial are: $-2, -10, -1$

5. Question

Obtain all zeros of the polynomial $f(x) = x^4 - 3x^3 - x^2 + 9x - 6,$ if two of its zeros are $-\sqrt{3}$ and $\sqrt{3}$.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha\ is\ a\ factor\ of\ f(x).$

Since -√3 and √3 are zeros of $f(x)$. Therefore $(x + \sqrt{3})$ and $(x - \sqrt{3})$ are factors of $f(x)$.

Now on dividing $f(x) = x^4 - 3x3 - x2 + 9x - 6\ by\ g(x) = (x - \sqrt{3})(x + \sqrt{3}) = x2 - 3$ to find other zeros.

$x^2 - 3 \overline{)\ x^4 - 3x^3 - x^2 + 9x - 6\ (}\ x^2 - 3x + 2$

$x^4 \quad - 3x^2$

$- \quad +$

$-3x^3 + 2x^2 + 9x - 6$

$-3x^3 \quad + 9x$

$+ \quad -$

$2x^2 \quad - 6$

$2x^2 \quad - 6$

$- \quad +$

0

By applying division algorithm, we have:

$x^4 - 3x3 - x2 + 9x - 6 = (x - \sqrt{3})(x + \sqrt{3})\ (x2 - 3x + 2)$

$x^4 - 3x3 - x2 + 9x - 6 = (x - \sqrt{3})(x + \sqrt{3})\ (x2 - 2x - x + 2)$

$x^4 - 3x3 - x2 + 9x - 6 = (x - \sqrt{3})(x + \sqrt{3})\ \{(x - 2)(x - 1)\}$

Hence, the zeros of the given polynomial are: $-\sqrt{3}, \sqrt{3}, 1, 2$

6. Question

Find all zeros of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6,$ it its two zeros are $-\frac{\sqrt{3}}{2}$ and $\frac{\sqrt{3}}{2}$.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha\ is\ a\ factor\ of\ f(x).$

Since $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$ are zeros of $f(x)$. Therefore $\left(x + \sqrt{\frac{3}{2}}\right)$ and $\left(x - \sqrt{\frac{3}{2}}\right)$ are factors of $f(x)$.

Now on dividing

$$f(x)=2x^4-2x3-7x2+3x+6\ by\ g(x)=\left(x+\sqrt{\frac{3}{2}}\right)\left(x-\sqrt{\frac{3}{2}}\right)=x2-\frac{3}{2}\Rightarrow\frac{1}{2}(2x^2-3)$$

to find other zeros.

$$\begin{array}{l}
2x^2-3\,\big)\ 2x^4-2x^3-7x^2+3x+6\ \big(\ x^2-x-2 \\
\qquad\quad 2x^4 \qquad\quad -3x^2 \\
\qquad\quad - \qquad\qquad + \\
\hline
\qquad\qquad -2x^3\ -4x^2+3x+6 \\
\qquad\qquad -2x^3 \qquad\quad +3x \\
\qquad\qquad + \qquad\qquad - \\
\hline
\qquad\qquad\qquad -4x^2 \qquad +6 \\
\qquad\qquad\qquad -4x^2 \qquad +6 \\
\qquad\qquad\qquad + \qquad\quad - \\
\hline
\qquad\qquad\qquad\qquad 0
\end{array}$$

By applying division algorithm, we have:

$$2x^4-2x3-7x2+3x+6=\left(x+\sqrt{\frac{3}{2}}\right)\left(x-\sqrt{\frac{3}{2}}\right)(x2-x-2)$$

$$2x^4-2x3-7x2+3x+6=\left(x+\sqrt{\frac{3}{2}}\right)\left(x-\sqrt{\frac{3}{2}}\right)(x2-2x+x-2)$$

$$2x^4-2x3-7x2+3x+6=\left(x+\sqrt{\frac{3}{2}}\right)\left(x-\sqrt{\frac{3}{2}}\right)\{(x-2)(x+1)\}$$

Hence, the zeros of the given polynomial are: $-\sqrt{\frac{3}{2}},\sqrt{\frac{3}{2}},-1,2$

7. Question

What must be added to the polynomial $f(x)=x^4+2x^3-2x^2+x-1$ so that the resulting polynomial is exactly divisible by x^2+2x-3?

Answer

$x-2$

By division Algorithm we have,

$$f(x)=\ g(x)\times q(x)+r(x)$$

$$f(x)-r(x)=\ g(x)\times q(x)$$

From above relation we got that if we add $-r(x)\ to\ f(x)$ then resulting polynomial is divisible by $g(x)$.

Now find the remainder when $f(x)$ is divisible by $g(x)$.

$$
\begin{array}{l}
x^2+2x-3 \overline{\big)\, x^4+2x^3-2x^2+x-1 \,\big(}\, x^2+1 \\
\qquad\qquad x^4+2x^3-3x^2 \\
\qquad\qquad - \quad - \quad + \\
\hline
\qquad\qquad\qquad\qquad x^2+x-1 \\
\qquad\qquad\qquad\qquad x^2+2x-3 \\
\qquad\qquad\qquad\qquad - \quad - \quad + \\
\hline
\qquad\qquad\qquad\qquad\qquad -x+2
\end{array}
$$

$$r(x) = -x + 2$$

Therefore $-r(x) = x - 2$ has to be added so that resulting polynomial is divisible by $g(x)$

8. Question

What must be subtracted from the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$?

Answer

Given : the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$

To find : What must be subtracted from the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$

Solution : Let g(x) be $x^2 - 4x + 3$ By applying division algorithm:

Dividend = Quotient× Divisor + Remainder

Dividend - Remainder = Quotient× Divisor

$$f(x) = q(x) \times g(x) + r(x)$$

$$f(x) - r(x) = q(x) \times g(x)$$

Now find the remainder when $f(x)$ is divisible by $g(x)$.

$$
\begin{array}{l}
x^2-4x+3 \overline{\big)\, x^4+2x^3-13x^2-12x+21 \,\big(}\, x^2+6x+8 \\
\qquad\qquad x^4-4x^3+3x^2 \\
\qquad\qquad - \quad + \quad - \\
\hline
\qquad\qquad\qquad 6x^3-16x^2-12x+21 \\
\qquad\qquad\qquad 6x^3-24x^2+18x \\
\qquad\qquad\qquad - \quad + \quad - \\
\hline
\qquad\qquad\qquad\qquad 8x^2-30x+21 \\
\qquad\qquad\qquad\qquad 8x^2-32x+24 \\
\qquad\qquad\qquad\qquad - \quad + \quad - \\
\hline
\qquad\qquad\qquad\qquad\qquad 2x-3
\end{array}
$$

$$r(x) = 2x - 3$$

Therefore $r(x) = 2x - 3$ has to be subtracted so that resulting polynomial is divisible by $g(x)$

9. Question

Find all the zeros of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeros are 2 and-2.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha\, is\, a\, factor\, of\, f(x).$

Since -2 and 2 are zeros of $f(x)$. Therefore $(x + 2)$ and $(x - 2)$ are factors of $f(x)$.

Now on dividing f(x) by (x - 2)(x + 2) = x^2 - 4 to find other zeros.

$$
\begin{array}{l}
x^2 - 4 \,\big)\, x^4 + x^3 - 34x^2 - 4x + 120 \,\big(\, x^2 + x - 30 \\
\quad x^4 \qquad - 4x^2 \\
\quad - \qquad\quad + \\
\hline
\quad x^3 - 30x^2 - 4x + 120 \\
\quad x^3 \qquad\quad - 4x \\
\quad - \qquad\qquad + \\
\hline
\quad -30x^2 \qquad + 120 \\
\quad -30x^2 \qquad + 120 \\
\quad + \qquad\qquad - \\
\hline
\qquad\quad 0
\end{array}
$$

By applying division algorithm, we have:

Hence, two other zeroes are -6 and 5

10. Question

Find all zeros of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha\, is\, a\, factor\, of\, f(x).$

Since -√2 and √2 are zeros of $f(x)$. Therefore $(x + \sqrt{2})$ and $(x - \sqrt{2})$ are factors of $f(x)$.

Now on dividing $f(x) = 2x^4 + 7x3 - 19x2 - 14x + 30\; by\; g(x) = (x - \sqrt{2})(x + \sqrt{2}) = x2 - 2$ to find other zeros.

$$
\begin{array}{l}
x^2 - 2 \,\big)\, 2x^4 + 7x^3 - 19x^2 - 14x + 30 \,\big(\, 2x^2 + 7x - 15 \\
\quad 2x^4 \qquad - 4x^2 \\
\quad - \qquad\quad + \\
\hline
\quad 7x^3 - 15x^2 - 14x + 30 \\
\quad 7x^3 \qquad\quad - 14x \\
\quad - \qquad\qquad + \\
\hline
\quad -15x^2 \qquad + 30 \\
\quad -15x^2 \qquad + 30 \\
\quad + \qquad\qquad - \\
\hline
\qquad\quad 0
\end{array}
$$

By applying division algorithm, we have:

$2x^4 + 7x3 - 19x2 - 14x + 30$ = $(x - \sqrt{2})(x + \sqrt{2})$ $(2x2 + 7x - 15)$

$2x^4 + 7x3 - 19x2 - 14x + 30$ = $(x - \sqrt{2})(x + \sqrt{2})$ $(x2 + 6x - 5x - 30)$

$2x^4 + 7x3 - 19x2 - 14x + 30$ = $(x - \sqrt{2})(x + \sqrt{2})$ $\{(x - \frac{3}{2})(x + 5)\}$

Hence, the zeros of the given polynomial are: $-\sqrt{2}, \sqrt{2}, -5, \frac{3}{2}$

11. Question

Find all the zeros of the polynomial, $2x^3 + x^2 - 6x - 3$ if two of its zeros are √3 and $-\sqrt{3}$.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha\ is\ a\ factor\ of\ f(x).$

Since -√3 and √3 are zeros of $f(x).$ Therefore $(x + \sqrt{3})$ and $(x - \sqrt{3})$ are factors of $f(x)$.

Now on dividing $f(x) = 2x3 + x2 - 6x - 3\ by\ g(x) = (x - \sqrt{3})(x + \sqrt{3}) = x2 - 3$ to find other zeros.

$$x^2 - 3 \,\overline{)\, 2x^3 + x^2 - 6x - 3 \,(}\; 2x + 1$$

$$\begin{array}{ll} 2x^3 & -6x \\ - & + \\ \hline x^2 & -3 \\ x^2 & -3 \\ - & + \\ \hline & 0 \end{array}$$

By applying division algorithm, we have:

$2x3 + x2 - 6x - 3 = (x - \sqrt{3})(x + \sqrt{3})\ (2x + 1)$

Hence, the zeros of the given polynomial are: $-\sqrt{3}, \sqrt{3}, \frac{-1}{2}$

12. Question

Find all the zeros of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of its zeros are $-\sqrt{2}$ and $\sqrt{2}$.

Answer

We know that if $x = \alpha$ is a zero of a polynomial then $x - \alpha\ is\ a\ factor\ of\ f(x).$

Since -√2 and √2 are zeros of $f(x).$ Therefore $(x + \sqrt{2})$ and $(x - \sqrt{2})$ are factors of $f(x)$.

Now on dividing $f(x) = x3 + 3x2 - 2x - 6\ by\ g(x) = (x - \sqrt{2})(x + \sqrt{2}) = x2 - 2$ to find other zeros.

$$x^2 - 2 \,\overline{)\, x^3 + 3x^2 - 2x - 6 \,(}\; x + 3$$

$$\begin{array}{ll} x^3 & -2x \\ - & + \\ \hline 3x^2 & -6 \\ 3x^2 & -6 \\ - & + \\ \hline & 0 \end{array}$$

By applying division algorithm, we have:

$x3 + 3x2 - 2x - 6 = (x - \sqrt{2})(x + \sqrt{2})\ (x + 3)$

Hence, the zeros of the given polynomial are: $-\sqrt{2}, \sqrt{2}, -3$

CCE - Formative Assessment

1. Question

Define a polynomial with real coefficients.

Answer

A polynomial is a mathematical expression containing a sum of powers in one or more variables multiplied by coefficients or we can say an expression of more than two algebraic terms that contain different powers of the same variable.But;

- Not divisible by a variable.
- A variable's exponents can only be 0,1,2,3,... etc.
- It can't have an infinite number of terms.

Example - $5xy^2 - 3x + 5y^3 - 3$

And a polynomial with real coefficients is a product of irreducible polynomials of first and second degrees or in simple words, a polynomial having only real numbers as coefficients is the real coefficient Polynomial.

2. Question

Define degree of a polynomial.

Answer

A degree in a polynomial function is the greatest exponent of that equation, which determines the most number of solutions that a function could have. Or in simple words the degree of a term is the sum of the exponents of the variables that appear in it, and thus is a non-negative integer.

Each equation contains from one to several terms, which are divided by numbers or variables with different exponents.

For Example,

$y = 3x^{13} + 5x^3$

As we can see it has two terms,

$3x^{13}$ and $5x^3$

And the degree of the polynomial is 13, and that's the highest degree of any term in the equation.

3. Question

Write the standard form of a linear polynomial with real coefficients.

Answer

As we know the condition for linear polynomial degree equals to 1

So,

$f(x) = ax^1 + b,$

Where,

$a \neq 0$

4. Question

Write the standard form of a quadratic polynomial with real coefficients.

Answer

To write the standard form of a quadratic polynomial with real coefficients;

Let's take a polynomial,

$f(x) = ax^2 + bx + c$,

As we know that the quadratic polynomial = 2

So,

$a \neq 0$

5. Question

Write the standard form of a cubic polynomial with real coefficients.

Answer

To write the standard form of a cubic polynomial with real coefficients

Let's take,

$f(x) = ax^3 + bx^2 + cx + d$,

Where $a \neq 0$

Because degree of a cubic polynomial is 3

6. Question

Define the value of a polynomial at a point.

Answer

Value of a polynomial at a point is described as the value obtained by the polynomial at that point in time.

For Example:

Polynomial $f(x) = x^3 - 2(1)^2 + 7(1) - 8$

Now the value of the polynomial at $x = 1$

So here we get;

$= 1 - 2 + 7 - 8$

$= - 2$

7. Question

Define zero of a polynomial.

Answer

A zero or we can say the root of a polynomial function is a number that, when put in for the variable, makes the function equal to zero. We use the Rational Zero Theorem to find all the zeros of a polynomial function and the possible rational roots of a polynomial equation.

8. Question

The sum and product of the zeros of a quadratic polynomial are $-\frac{1}{2}$ and — 3 respectively. What is the quadratic polynomial?

Answer

Given,

Sum of zeroes (α + β) = −(1/2)

Product of zeroes (αβ) = −3As we know that the quadratic polynomial with zeroes α, β is given by;

$x^2 - (\alpha + \beta)x + (\alpha\beta) = x^2 - [-(1/2)]x + (-3) = x^2 + (1/2)x - 3$

9. Question

Write the family of quadratic polynomials having $-\frac{1}{4}$ and 1 as its zeros.

Answer

Given,

– 1/4 and 1 are the zeros

So,

k[(x + 1/4)(x – 1)]

$k[x^2 - 1x + 1/4x - 1/4]$

$f(x) = k[x^2 - 3/4\ x - 1/4]$

Where k is any non-zero real number.

10. Question

If the product of zeros of the quadratic polynomial $f(x) = x^2 - 4x + k$ is 3, find the value of k.

Answer

Given,

Let the quadratic polynomial be $f(x) = x^2 - 4x + k$Product of the zeroes of the quadratic polynomial = 3Now,

Product of the zeroes = constant term/coefficient of x^2

⇒ 3 = k/1∴ k = 3

11. Question

If the sum of the zeros of the quadratic polynomial f (x) = kx^2 — 3x + 5 is 1, write the value of k.

Answer

Given,

The quadratic polynomial f(x) = kx^2 – 3x + 5Now,

Let two zeroes be a and 1 – a

∵ Sum of zeroes = 1 and also 1 – a > 0 ⇒ a < 1

Therefore;

Sum of the zeroes = 3/k

⇒ a + (1 – a) = 3/k

⇒ 1 = 3/k

⇒ k = 3

12. Question

In Fig. 2.17, the graph of a polynomial p(x) is given. Find the zeros of the polynomial.

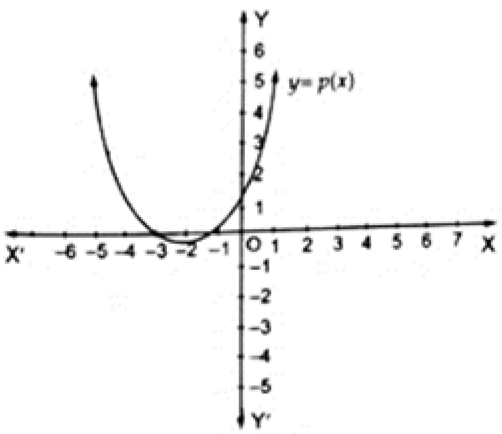


Fig. 2.17

Answer

In figure 2.17, the polynomial has 2 zeroes because the graph cuts x-axis at 2 points i.e. x = – 3 and x = – 1.

13. Question

The graph of a polynomial y = f (x), shown in Fig. 2.18. Find the number of real zeros of f (x).

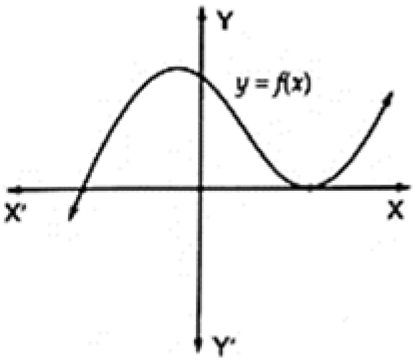


Fig. 2.18

Answer

In figure 2.18, the polynomial has 3 real zeroes because the graph cuts axises at three points.

14. Question

The graph of the polynomial $f(x) = ax^2 + bx + c$ is as shown below (Fig. 2.19). Write the signs of 'a' and $b^2 - 4ac$.

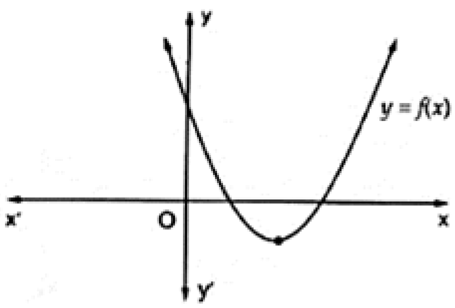


Fig. 2.19

Answer

The signs of a will be positive as $a > 0$ and,

The signs of $b^2 - 4ac$ will be positive as $b^2 - 4ac > 0$

15. Question

The graph of the polynomial $f(x) = ax^2 + box + c$ is as shown in Fig. 2.20. Write the value of $b^2 - 4ac$ and the number of real zeros of f (x).

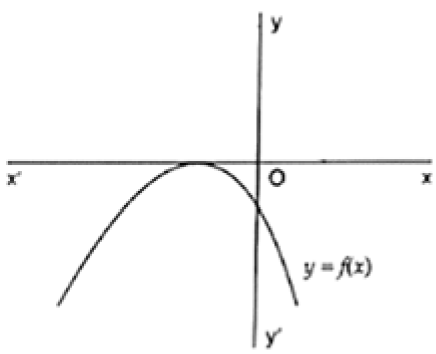


Fig. 2.20

Answer

$b^2 - 4ac = 0$, Two

The given quadratic equation touches the x-axis at only one point.

The root of the quadratic equation is equal and real because if the quadratic equation has two distinct roots, then the graph touches the x-axis at two points.

As we know that the roots are real and equal if the value of discriminant is zero,

So,

$b^2 - 4ac = 0$

16. Question

In Q. No. 14, write the sign of c.

Answer

The y-intercept of the equation is c,

So, the signs of will be positive as $c > 0$

17. Question

In Q. No. 15, write the sign of c.

Answer

The y-intercept of the equation is c,

So, the signs of will be negative as $c < 0$

18. Question

The graph of a polynomial/ (x) is as shown in Fig. 2.21. Write the number of real zeros of f (x).

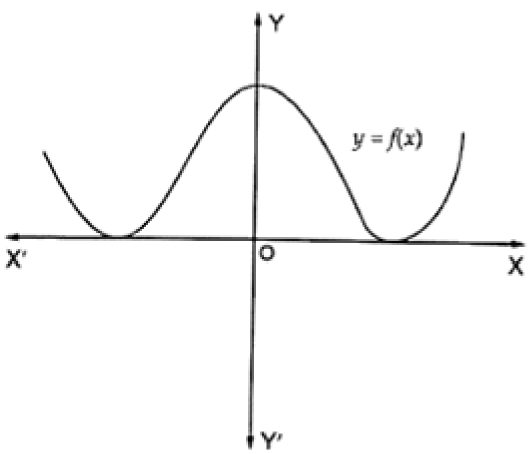


Fig. 2.21

Answer

In the figure 2.21,

The polynomial has 4 real zeroes because the graph cuts x-axis at four points.

19. Question

If x = 1 is a zero of the polynomial f (x) = $x^3 - 2x^2 + 4x + k$, write the value of k.

Answer

Given,

$f(x) = x^3 - 2x^2 + 4x + k$

x = 1 is a zero,

So,

Keeping x = 1 we get,

$f(1) = 1^3 - 2(1)^2 + 4\times1 + k$

f(1) = 1 − 2 + 4 + k

f(1) = 5 − 2 + k

f(1) = 3 + k

− 3 = k

So we get,

k = − 3

20. Question

State division algorithm for polynomials.

Answer

The polynomial long division is an algorithm for dividing a polynomial by another polynomial of the same or lower degree; it is a generalized version of the familiar arithmetic technique called long division. It can be done manually because it separates a complex division problem into smaller ones.

Let's take the Example:

f(x) and g(x) are two polynomials with,

$g(x)\neq0$,

Now we can find the polynomials p(x) and q(x) such that,

$f(x) = p(x) \times g(x) \times q(x)$

Where q(x) = 0 or degree of q(x) < is degree of g(x).

The result is;

Dividend = Quotient × Divisor + Remainder

This is known as the Division Algorithm for polynomials.

21. Question

Give an example of polynomials f(x), g(x), q(x) and r(x) satisfying f(x) = g(x) .q(x)+ r(x), where degree r (x) = 0.

Answer

Let $f(x) = x^3 + x^2 + x + 1$

$g(x) = x + 2$

$q(x) = x^2 - x + 3$

$r(x) = -5$

Now, f(x) = g(x) .q(x)+ r(x)

Here,

L.H.S = $x^3 + x^2 + x + 1$

And, R.H.S = $[(x + 2) \times (x^2 - x + 3)] + (-5)$

$= x^3 + 2x^2 - x^2 - 2x + 3x + 6 - 5$

$= x^3 + x^2 - 2x + 3x + 6 - 5$

$= x^3 + x^2 + x + 1$

= R.H.S

∴ the given set of polynomials satisfies the equation: f(x) = g(x) .q(x)+ r(x)

22. Question

Write a quadratic polynomial, sum of whose zeros is 2√3 and their product is 2.

Answer

Given,

Sum of zeros = 2√3 and

Product = 2

As we know;

$f(x) = x^2 + (-\text{Sum of zeros})x + (\text{Product of zeros})$

So,

$f(x) = x^2 - 2\sqrt{3}x + 2$

23. Question

If fourth degree polynomial is divided by a quadratic polynomial, write the degree of the remainder.

Answer

Degree of reminder is less than the degree of divisor so the degree of reminder could be one or zero depending on the quadratic polynomial.

24. Question

If $f(x) = x^3 + x^2 - ax + b$ is divisible by $x^2 - x$ write the values of a and b.

Answer

$a = 2, b = 0$

Given,

A polynomial $f(x) = x^3 + x^2 - ax + b$

Which is divisible by $x^2 - x$

Now,

$x^2 - x = (x - 1) = (x - 0)(x - 1)$

$(x - 0)$ and $(x - 1)$ are factors of polynomial f(x)

$\Rightarrow f(0) = 0$

$\Rightarrow 0^3 + 0 - a \times 0 + b = 0$

$\Rightarrow b = 0$

And f(1)

$1^3 + 1 - a \times 1 + b = 0$

$\Rightarrow 2 - a = 0$

$\Rightarrow a = 2$

$\therefore a = 2$ and $b = 0$

25. Question

If a – b, a and a + b are zeros of the polynomial $f(x) = 2x^3 - 6x^2 + 5x - 7$, write the value of a.

Answer

Given;

A polynomial $f(x) = 2x^3 - 6x^2 + 5x - 7$

And zeroes are a – b, a and a + b,

Let's take $a = \alpha$

$b = \beta$

$c = y$

As we know that,

$\alpha + \beta + y = - b/a$

$(a - b) + a + (a + b) = - (- 6)/2$

$3a = 3$

$a = 1$

So, the value of $a = 1$

26. Question

Write the coefficients of the polynomial $p(z) = z^5 - 2z^2 + 4$.

Answer

Given,

Polynomial $p(z) = z^5 - 2z^2 + 4$

Now first break the given equation,

We get,

Coefficient of $z^5 = 1$

Coefficient of $z^4 = 0$

Coefficient of $z^3 = 0$

Coefficient of $z^2 = - 2$

Coefficient of $z = 0$

Constant term $= 4$

Therefore 1, 0, 0, − 2, 0, 4 are coefficients of the polynomial p(z)

27. Question

Write the zeros of the polynomial $x^2 - x - 6$.

Answer

Given,

Polynomial $= X^2 - x - 6 = 0x^2 - 3x + 2x - 6 = 0x(x - 3) + 2(x - 3) = 0(x + 2)(x - 3) = 0$So − 2 and 3 are the zeros of the given polynomial

28. Question

If $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$, find a.

Answer

Given,

$2x^2 + 2ax + 5x + 10$

Factor = (x + a)

Now by using factor theoram we get,

$f(-a) = 2a^2 - 2a^2 - 5a + 10 = 0$

$-5a + 10 = 0$

a = 2

Therefore the value of a = 2.

29. Question

For what value of k, − 4 is a zero of the polynomial $x^2 - x - (2k + 2)$?

Answer

Given,

A polynomial $x^2 - x - (2k + 2)$

− 4 is a zero of the given polynomial.

As − 4 is the zero,so at x = -4, the value of the polynomial $x^2 - x - (2k + 2)$ will be 0.

So we get,

$\Rightarrow x^2 - x - (2k + 2) = 0$

$\Rightarrow (-4)^2 - (-4) - (2k + 2) = 0$

$\Rightarrow 16 + 4 - (2k + 2) = 0$

$\Rightarrow 20 - (2k + 2) = 0$

$\Rightarrow -(2k + 2) = -20$

$\Rightarrow 2k + 2 = 20$

$\Rightarrow 2k = 20 - 2$

$\Rightarrow 2k = 18$

$\Rightarrow k = 18/2 = 9$

30. Question

If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a - 1)x - 1$, then find the value of a.

Answer

Given,

$p(x) = ax^2 - 3(a - 1)x - 1$

Zero is 1

Now,

$p(1) = a(1)^2 - 3(a - 1) \times 1 - 1 = 0$

$p(1) = a - 3a + 3 - 1 = 0$

$p(1) = - 2a + 2 = 0$

$= - 2a = - 2$

$= a = - 2/ - 2 = 1$

So the value of a = 1

31. Question

If α, β are the zeros of a polynomial such that α + β = − 6 and αβ = − 4, then write the polynomial.

Answer

For any polynomial, $f(x) = ax^2 + bx + c$

The sum of zeroes, is given as -b/a

& the product of zeroes is given as c/a

Here, the sum of zeroes is given as -6 = -6/1

$\Rightarrow -\frac{6}{1} = -\frac{b}{a}$

⇒ b = 6 & a = 1

Also, product of zeroes = c/a

$\Rightarrow -4 = \frac{c}{a}$

⇒ c = -4

& a = 1

∴ the polynomial is $f(x)= x^2 +6x-4$

32. Question

If α, β are the zeros of the polynomial $2y^2 + 7y + 5$, write the value of α + β + αβ.

Answer

Given,

Polynomial $2y^2 + 7y + 5$

Zeroes = a, 13

Here a = 2

b = 7

c = 5

Lets take α and β are two zero,

So sum of the zeroes will be,

$\alpha + \beta = - b/a = - 7/2$

Product of the zeroes = $\alpha . \beta = c/a = 5/2$

Now put the values,

$\alpha + \beta + \alpha\beta = (\alpha + \beta)\ \alpha\beta$

$= - 7/2 + 5/2 = - 2/2 = - 1$

33. Question

For what value of k, is 3 a zero of the polynomial $2x^2 + x + k$?

Answer

Given,

Polynomial $2x^2 + x + k$

Zero = 3

By putting x = 3

We get,

$p(x) = 2x^2 + x + k = 0$

$p(3) = 2(3)^2 + 3 + k = 0$

$p(3) = 18 + 3 + k = 0$

$= 21 + k = 0$

$k = - 21$

Hence the answer is − 21

34. Question

For what value of k, is − 3 a zero of the polynomial $x^2 + 11x + k$?

Answer

Given,

Polynomial $x^2 + 11x + k$

Zero of the polynomial = − 3

As we have zero,

$f(x) = x^2 + 11x + k = 0$

By putting x = − 3

$f(-3) = -3^2 + 11 \times 3 + k = 0$

$f(-3) = 9 + (-33) + k = 0$

$f(-3) = -24 + k = 0$

$f(-3) = k = 24$

So we have the value of k = 24

35. Question

For what value of k, is − 2 a zero of the polynomial $3x^2 + 4x + 2k$?

Answer

Given,

Polynomial $3x^2 + 4x + 2k$

Zero of the polynomial = − 2

As we have zero,

$f(x) = 3x^2 + 4x + 2k = 0$

By putting x = − 2

$f(-2) = 3(-2)^2 + 4(-2) + 2k = 0$

$f(-2) = 12 - 8 + 2k = 0$

$= 4 + 2k = 0$

$= 2k = -4$

$k = -4/2 = -2$

So we have the value of k = − 2

36. Question

If a quadratic polynomial f (x) is factorizable into linear distinct factors, then what is the total number of real and distinct zeros of f(x)?

Answer

Given,

Quadratic polynomial f(x) is factorizable into linear distinct factors;

So,

Let $f(x) = (x - a)(x - b)$, where $a \neq b$

If a, b are the element of R,

Then f(x) must be having two real and distinct zeroes.

37. Question

If a quadratic polynomial f (x) is a square of a linear polynomial, then its two zeroes are coincident. (True/False)

Answer

True

Lets take,

$f(x) = x^2 - 4x + 4$

$= (x - 2)^2$

$= [g(x)]^2$ ………….. [g(x) = (x – 2) is a linear polynomial]

Zero of g(x) is 2,

So,

Zeroes of f(x) are 2 and 2.

So we can say that zeroes of f(x) are coincident.

38. Question

If a quadratic polynomial f(x) is not factorizable into linear factors, then it has no real zero. (True/False)

Answer

True

Let a polynomial $f(x) = x^2 + 9$

$f(x) = (x + 3i)(x - 3i)$

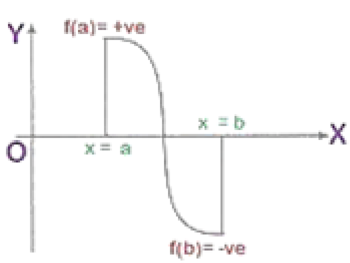
So, the zeroes are always imaginary and not real.

39. Question

If f(x) is a polynomial such that f(a)f(b)< 0, then what is the number of zeros lying between a and b?

Answer

Let's first draw the figure,



As we can see in the figure the number of zeroes can be 1 or 3.

So the least numbers of zeroes lying between a and b is 1.

40. Question

If graph of quadratic polynomial $ax^2 + bx + c$ cuts positive direction of y-axis, then what is the sign of c?

Answer

Positive

If quadratic polynomial $ax^2 + bx + c$ cuts positive direction of y-axis, then it means that the value of y is positive when x = 0 [∵ at y-axis, x = 0]

Now

y = c at x = 0 and y is positive.

So, c is positive.

41. Question

If the graph of quadratic polynomial $ax^2 + bx + c$ cuts negative direction of y-axis, then what is the sign of c?

Answer

Negative

If quadratic polynomial $ax^2 + bx + c$ cuts Negative direction of y-axis, then it means that the value of y is negative when x = 0 [∵ at y-axis, x = 0]

Now,

y = c at x = 0 and y is Negative.

So, c is Negative.

1. Question

If α, β are the zeros of the polynomial f (x) = $x^2 + x + 1$, then $\frac{1}{\alpha}+\frac{1}{\beta}$ =

A. 1

B. − 1

C. 0

D. None of these

Answer

Given,

$f(x) = x^2 + x + 1$

α = a

β = b

c = c

$\alpha + \beta = -\, b/a$

$= -\, 1/1$

$= -\, 1$ …….. Equ.(i)

$\alpha\beta = c/a = 1$ ….. Equ.(ii)

Dividing (i) from (ii)

We get,

$-\, 1/1 = -\, 1$

So,

$1/\alpha + 1/\beta = -\, 1$

2. Question

If α, β are the zeros of the polynomial $p(x) = 4x^2 + 3x + 7$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

A. $\frac{7}{3}$

B. $-\frac{7}{3}$

C. $\frac{3}{7}$

D. $-\frac{3}{7}$

Answer

Given,

$p(x) = 4x^2 + 3x + 7$

$\alpha + \beta = -\, b/a$

$= -\, 3x/4x^2 = -\, 3/4x$ ………. (i)

$\alpha\beta = c/a$

$= 7/4x^2$ …………. (ii)

Dividing (i) from (ii),

We get,

$$\frac{\frac{3}{4x}}{\frac{7}{x^2}} = \frac{-3 \times 4x^2}{4x \times 7}$$

$$= -\frac{3}{7}x$$

So,

1/α + 1/β = – 3/7

3. Question

If one zero of the polynomial f (x) = $(k^2 + 4)\,x^2 + 13x + 4k$ is reciprocal of the other, then k =

A. 2

B. – 2

C. 1

D. – 1

Answer

Given;

f(x) = $(k^2 + 4)\,x^2 + 13x + 4k$,

One zero of the polynomial is reciprocal of the other,

Let a be the one zero,

∴ The other zero will be 1/a

As we know that,

Product of the zeros = c/a = $4k/k^2 + 4$

∴ $4k/k^2 + 4 = 1$

⇒ $4k = k^2 + 4$

⇒ $k^2 + 4 - 4k = 0$

⇒ $(k - 2)^2 = 0$

⇒ k = 2

So the value of k is 2

4. Question

If the sum of the zeros of the polynomial f(x) = $2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of k is

A. 2

B. 4

C. − 2

D. − 4

Answer

Given,

f(x) = $2x^3 - 3kx^2 + 4x - 5$

Sum of the zeros of the polynomial = 6

Let x, y and z be the zeroes than,

x + y + z = 6……Equation (i)

So,

x + y + z = − b/a

= − (− 3k)/2

From Eq. (i) we get,

3k/2 = 6

k/2 = 6/3

k = 2×2 = 4

5. Question

If α and β are the zeros of the polynomial f(x) = $x^2 + px + q$, then a polynomial having $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is its zeros is

A. $x^2 + qx + p$

B. $x^2 - px + q$

C. $qx^2 + px + 1$

D. $px^2 + qx + 1$

Answer

Given: If α and β are the zeros of the polynomial f(x) = $x^2 + px + q$,

To find: a polynomial having $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is its zeros is

Solution:

f(x) = $x^2 + px + q$

We know, $sum\ of\ zeroes = -\frac{coefficient\ of\ x}{coefficient\ of\ x^2}$

$$product\ of\ zeroes = \frac{constant\ term}{coefficient\ of\ x^2}$$

Since α, β are zeroes of given polynomial,

⇒ α + β = − p

and αβ = q

Let S and P denote respectively the sum and product of zeroes of the required polynomial,So,

$$S = \frac{1}{\alpha} + \frac{1}{\beta}$$

$$\Rightarrow S = \frac{\beta + \alpha}{\alpha\beta} \quad \text{...... (1)}$$

And $P = \frac{1}{\alpha} \times \frac{1}{\beta}$ ...... (2)

Put the values of α + β and αβ in (1) and (2) to get,

$$\Rightarrow S = \frac{-p}{q}$$

And $P = \frac{1}{q}$

We know equation having 2 zeroes is of form,k (x^2 - (sum of zeroes) x + product of zeroes)

For a polynomial having $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is its zeros the equation becomes,

$x^2 + p/q\ x + 1/q = 0$

So here we get,

$g(x) = qx^2 + px + 1$

6. Question

If α, β are the zeros of polynomial $f(x) = x^2 - p(x + 1) - c$, then (α + 1) (β + 1) =

A. c − 1

B. 1 − c

C. c

D. 1 + c

Answer

Given,

$f(x) = x^2 - p(x + 1) - c$

α and β are the zeros

So,

$f(x) = x^2 - p(x + 1) - c$

$= x^2 - px - (p + c)$

As

$(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1$

$= - p - c + p + 1$

$= 1 - c$

7. Question

If α, β are the zeros of the polynomial $f(x) = x^2 - p(x + 1) - c$ such that $(\alpha + 1)(\beta + 1) = 0$, then c =

A. 1

B. 0

C. − 1

D. 2

Answer

Given

$f(x) = x^2 - p(x + 1) - c$

α and β are the zeros

Then,

$f(x) = x^2 - p(x + 1) - c$

$= x^2 - px - (p + c)$

As

$(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1$

$= - p - c + p + 1$

$= 1 - c$

So, the value of c,

$c = 1$

8. Question

If $f(x) = ax^2 + bx + c$ has no real zeros and $a + b + c < 0$, then

A. $c = 0$

B. $c > 0$

C. $c < 0$

D. None of these

Answer

Given;

$f(x) = ax^2 + bx + c$ has no real zeroes, and $a + b + c<0$

Suppose $a = -1$,

$b = 1$,

$c = -1$

Then $a + b + c = -1$,

$b^2 - 4ac = -3$

Therefore it is possible that c is less tha zero.

Suppose $c = 0$

Then $b^2 - 4ac = b^2 \geq 0$

So,

f(x) has at least one zero.

Therefore c cannot equal zero.

Suppose $c > 0$.

It must also be true that $b^2 \geq 0$

Then,

$b^2 - 4ac < 0$ only if $a > 0$.

Therefore,

$a + b + c < 0$.

$-b > a + c > 0$

$b^2 > (a + c)^2$

$b^2 > a^2 + 2ac + c^2$

$b^2 - 4ac > (a - c)^2 \geq 0$

As we know that the discriminant can't be both greater than zero and less than zero,

So, C can't be greater than zero.

9. Question

If the diagram in Fig. 2.22 shows the graph of the polynomial f (x) = $ax^2 + bx + c$, then

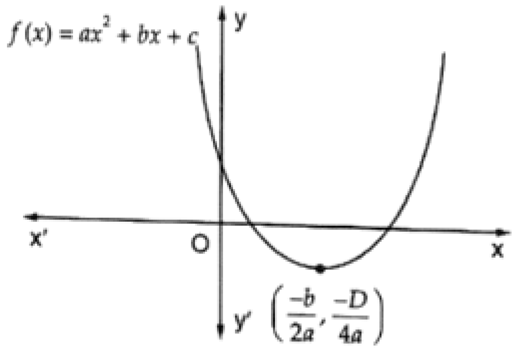


Fig. 2.22

A. a > 0, b<0 andc > 0

B. a < 0, b <0 and c< 0

C. a < 0, b > 0 and c > 0

D. a < 0, b > 0 and c < 0

Answer

As seen from the graph,

The parabola cuts the graph at two points on the positive x-axis.

Hence, both the roots are positive.

Now, for a polynomial, the sum of roots is given as:

$\alpha + \beta = -b/a$

∴ the sum will be positive as the roots are positive.

Also, the product of roots = c/a has to be positive too.

⇒ a is positive.

Now, since a is positive, therefore for the sum of roots to be negative, b has to be negative.

⇒ a > 0, b< 0 & c > 0.

Therefore, option (a) is correct.

10. Question

Figure 2.23 shows the graph of the polynomial $f(x) = ax^2 + bx + c$ for which

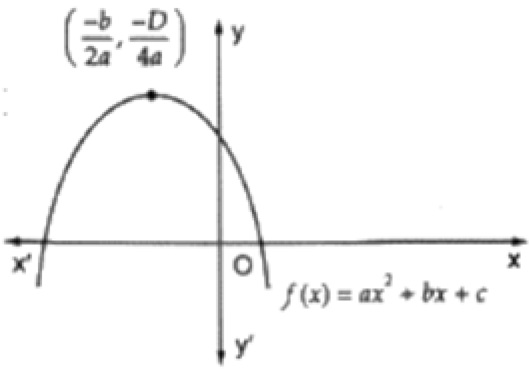


Fig. 2.23

A. $a < 0$, $b > 0$ and $c > 0$

B. $a < 0$, $b < 0$ and $c > 0$

C. $a < 0$, $b < 0$ and $c < 0$

D. $a > 0$, $b > 0$ and $c < 0$

Answer

As seen from the graph,

The parabola cuts the graph at two points on the x axis.

One root is positive & one root is negative.

Now, for a polynomial, the sum of roots is given as:

$\alpha + \beta = -b/a$

Also, the product of roots = c/a is positive.

Because c is positive,

⇒ a is negative & b is negative.

Therefore, option (b) is correct.

11. Question

If the product of zeros of the polynomial $f(x) = ax^3 - 6x^2 + 11x - 6$ is 4, then a =

A. $\frac{3}{2}$

B. $-\frac{3}{2}$

C. $\frac{2}{3}$

D. $-\frac{2}{3}$

Answer

Given;

Polynomial = $ax^3 - 6x^2 + 11x - 6$

Let a,b,c be the zeroes of $f(x) = ax^3 - 6x^2 + 11x - 6$

Product of the zeroes = − (constant term)/coefficient of x^3

4 = − (− 6)/a

4 = 6/a

4a = 6

a = 6/4

a = 3/2

12. Question

If zeros of the polynomial $f(x) = x^3 - 3px^2 + qx - r$ are in A.P., then

A. $2p^3 = pq - r$

B. $2p^3 = pq + r$

C. $p^3 = pq - r$

D. None of these

Answer

Given;

$f(x) = x^3 - 3px^2 + qx - r$

Let a, b and c be the zeroes of the polynomial $x^3 - 3px^2 + qx - r$

a + b + c = 3 p

ab + bc + ac = q

abc = r

a + b = 2b ⇒ b = p ⇒ a + c = 2p

b(a + c) + ac = q

$\Rightarrow 2p^2 + ac = q$

$\Rightarrow ac = r/p$

$\therefore 2p^2 + r/p = q$

$2p^3 = pq - r$

13. Question

If the product of two zeros of the polynomial f (x) = $2xZ^3 + 6x^2 - 4x + 9$ is 3, then its third zero is

A. $\frac{3}{2}$

B. $-\frac{2}{3}$

C. $\frac{9}{2}$

D. $-\frac{9}{2}$

Answer

Given;

$f(x) = 2x^3 + 6x^2 - 4x + 9$

Product of the two zeros = 3

a = 2,

b = 6

c = − 4

d = 9

Let the zeros of f(x) be p, q, r

Product of the zeroes = − d/a

p×q×r = − 9/2

3r = − 9/2 … (given p×q = 3)

r = − 3/2

Hence,

Third zero = − 3/2

14. Question

If the polynomial $f(x) = ax^3 + bx - c$ is divisible by the polynomial $g(x) = x^2 + bx + c$, then ab =

A. 1

B. $\frac{1}{c}$

C. −1

D. $-\frac{1}{c}$

Answer

Given;

$f(x) = ax^3 + bx - c$ is divisible by $g(x) = x^2 + bx + c$

Now by division Method,

$$
\begin{array}{r|l}
 & ax - ab \\
\hline
x^2 + bx + c & ax^3 + bx - c \\
 & ax^3 + abx^2 + acx \\
 & - \quad - \quad - \\
\hline
 & -abx^2 + (b - ac)x - c \\
 & -abx^2 - ab^2x - abc \\
 & + \quad + \quad + \\
\hline
 & (b - ac + ab^2)x + abc - c
\end{array}
$$

As f(x) is divisible by g(x), then remainder must be 0,

i.e.

$(ab^2 - ac + b)x + c(ab - 1) = 0$

$\Rightarrow (ab^2 - ac + b)x = 0$ and $c(ab - 1) = 0$

$\Rightarrow ab^2 - ab + b = 0$ ($\because x \neq 0$) and $ab - 1 = 0$ ($\because c \neq 0$)

$\Rightarrow ab - 1 = 0$

$\Rightarrow ab = 1$

15. Question

In Q. No. 14, c =

A. b

B. 2b

C. $2b^2$

D. − 2b

Answer

Given: $f(x) = ax^3 + bx - c$ is divisible by $g(x) = x^2 + bx + c$

To find: The value of c.

Solution:

Since $f(x) = ax^3 + bx - c$ is divisible by $g(x) = x^2 + bx + c$

Now by division Method first we have to calculate the value of ab,

$$\begin{array}{r|l} & ax - ab \\ x^2 + bx + c & ax^3 + bx - c \\ & ax^3 + abx^2 + acx \\ & - \quad - \quad - \\ \hline & -abx^2 + (b - ac)x - c \\ & -abx^2 - ab^2x - abc \\ & + \quad + \quad + \\ \hline & (b - ac + ab^2)x + abc - c \end{array}$$

As f(x) is divisible by g(x), then remainder must be 0,

i.e.

$(ab^2 - ac + b)x + abc - c = 0$

$\Rightarrow (ab^2 - ac + b)x + c(ab - 1) = 0$

$\Rightarrow (ab^2 - ac + b)x = 0$ and $c(ab - 1) = 0$

$\Rightarrow ab^2 - ac + b = 0$ ($\because x \neq 0$) and $ab - 1 = 0$ ($\because c \neq 0$)

$\Rightarrow ab - 1 = 0$

$\Rightarrow ab = 1$

Now take;

$ab^2 - ac + b = 0$

$\Rightarrow (ab)b - ac + b = 0$

Using ab = 1 we get;

$\Rightarrow b - ac + b = 0$

$\Rightarrow 2b - ac = 0 \Rightarrow c = 2b/a$ ... (1)Multiply and divide RHS of (1) by b

$\Rightarrow c = 2b/a = 2b^2/ab$

As ab = 1

So we get;

$c = 2b^2$

16. Question

If one root of the polynomial $f(x) = 5x^2 + 13x + k$ is reciprocal of the other, then the value of k is

A. 0

B. 5

C. $\frac{1}{6}$

D. 6

Answer

Given;

$f(x) = 5x^2 + 13x + k$

Let suppose roots are R and 1/R

Product of the roots = C/R = R×1/R

R×1/R = k/5

1 = k/5

So,

Here k = 5

17. Question

If α, β, γ are the zeros of the polynomial $f(x) = ax^3 + bx^2 + cx + d$, then $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} =$

A. $-\frac{b}{d}$

B. $\frac{c}{d}$

C. $-\frac{c}{d}$

D. $-\frac{c}{a}$

Answer

Given,

$f(x) = ax^3 + bx^2 + cx + d$,

As we know,

$\alpha + \beta + \gamma = b/a$

$\alpha\beta + \beta\gamma + \gamma\alpha = -c/a$ ……………(i)

$\alpha\beta\gamma = d/a$ ……………….. (ii)

Now,

Divide (i) by (ii)

We get

$1/\gamma + 1/\alpha + 1/\beta = -c/d$

18. Question

If α, β, γ are the zeros of the polynomial $f(x) = ax^3 + bx^2 + cx + d$, then $\alpha^2 + \beta^2 + \gamma^2 =$

A. $\frac{b^2 - ac}{a^2}$

B. $\frac{b^2 - 2ac}{a}$

C. $\frac{b^2 + 2ac}{b^2}$

D. $\frac{b^2 + 2ac}{a^2}$

Answer

$f(x) = ax^3 + bx^2 + cx + d$,

Then the roots are : α , β and γ

Now, sum of roots:

$\alpha + \beta + \gamma = - b/a$ ….(i)

The product of zeroes,

α × β × γ = -d/a….(ii)

And,

αβ + βγ + γα = c/a…(iii)

Now,

$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2\alpha\beta + 2\beta\gamma + 2\gamma\alpha$

$\Rightarrow \left(-\frac{b}{a}\right)^2 = a^2 + \beta^2 + \gamma^2 + 2\left(\frac{c}{a}\right)$

$\Rightarrow \frac{b^2}{a^2} - \frac{2c}{a} = a^2 + \beta^2 + \gamma^2$

$\Rightarrow a^2 + \beta^2 + \gamma^2 = \frac{(b^2-2ac)}{a}$

19. Question

If α, β, γ are the zeros of the polynomial $f(x) = x^3 - px^2 + qx - r$, then $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} =$

A. $\frac{r}{p}$

B. $\frac{p}{r}$

C. $-\frac{p}{r}$

D. $-\frac{r}{p}$

Answer

Given,

$f(x) = x^3 - px^2 + qx - r$

α, β and γ are the Zeros,

α + β + γ = − p

αβ + βγ + γα = − q

αβγ = − r

1/αβ + 1/βγ + 1/γα = p/r

20. Question

If α, β are the zeros of the polynomial $f(x) = ax^2 + bx + c$, then $\frac{1}{\alpha^2} + \frac{1}{\beta^2} =$

A. $\frac{b^2 - 2ac}{a^2}$

B. $\frac{b^2 - 2ac}{c^2}$

C. $\frac{b^2 + 2ac}{a^2}$

D. $\frac{b^2 + 2ac}{c^2}$

Answer

Given

$f(x) = ax^2 + bx + c$

α, β are the Zeros

$\alpha + \beta = -b/a$

$\alpha\beta = c/a$

$1/\alpha + 1/\beta = \frac{-\frac{b}{a}}{\frac{c}{a}}$

$1/\alpha^2 + 1/\beta^2 = (1/\alpha + 1/\beta)^2 - 2/\alpha\beta$

$= (-b/c)^2 - 2a/c$

$= \frac{b^2 - 2ac}{c^2}$

21. Question

If two of the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$ are each equal to zero, then the third zero is

A. $\frac{-d}{a}$

B. $\frac{c}{a}$

C. $\frac{-b}{a}$

D. $\frac{b}{a}$

Answer

Given,

$ax^3 + bx^2 + cx + d$

By Putting x = 0

0 + d = 0

d = 0

$ax^3 + bx^2 + cx + d = 0$

$x(ax^2 + bx + c) = 0$

Put x = 0

c = 0

$ax^2 + bx = 0$

x(ax + b) = 0

Hence,

x = – b/a

22. Question

If two zeros of $x^3 + x^2 - 5x - 5$ are $\sqrt{5}$ and $-\sqrt{5}$, then its third zero is

A. 1

B. – 1

C. 2

D. – 2

Answer

Given,

$x^3 + x^2 - 5x - 5$

Zeros = $\sqrt{5}$ and – $\sqrt{5}$

$x^2 - 5$ is a root

$$x + 1$$
$$x^2 - 5 \overline{)\, x^3 - x^2 - 5x - 5}$$
$$-x^3 - 5x$$
$$+$$
$$x^2 - 5$$

$(x^2 - 5)(x + 1) = 0$

$x = -1$

So,

The third zero is − 1

23. Question

The product of the zeros of $x^3 + 4x^2 + x - 6$ is

A. − 4

B. 4

C. 6

D. − 6

Answer

Given,

$p(x) = x^3 + 4x^2 + x - 6$

As

$a = 1$

$b = 4$

$c = 1$

$d = -6$

So the product of the zeros will be;

$= -d/a$

$= -(-6)/1$

$= 6$

24. Question

What should be added to the polynomial $x^2 - 5x + 4$, so that 3 is the zero of the resulting polynomial?

A. 1

B. 2

C. 4

D. 5

Answer

Given;

$p(x) = x^2 - 5x + 4$

3 is the zero of the resulting polynomial

So we have;

$p(3) = (3)^2 - 5\times3 + 4$

$p(3) = 9 - 15 + 4$

$p(3) = - 2$

As − 2 is the remainder,

So 2 should be added to the given polynomial to get 3 as the zero of the resulting polynomial.

We can also verify the answer;

By adding 2 to the given polynomial we get;

$P(x) = x^2 - 5x + 4 + 2$

$P(x) = x^2 - 5x + 6$

$P(3) = 9 - 15 + 6$

$P(3) = 15 - 15$

$P(3) = 0$

Hence proved that 2 is the right answer to get 3 as the zero of resulting polynomial.

25. Question

What should be subtracted to the polynomial $x^2 - 16x + 30$, so that 15 is the zero of the resulting polynomial?

A. 30

B. 14

C. 15

D. 16

Answer

Given,

$p(x) = x^2 - 16x + 30$

15 is the zero of the resulting polynomial,

So we get;

$p(15) = (15)^2 - 16 \times 15 + 30$

$p(15) = 225 - 240 + 30$

$p(15) = 15$

∴ 15 should be subtracted from the given polynomial to get 15 as the zero of the resulting polynomial.

We can also verify the answer;

By adding 2 to the given polynomial we get;

$P(x) = x^2 - 16x + 30 - 15$

$P(x) = x^2 - 16x + 15$

$P(15) = 225 - 240 + 15$

$P(15) = 240 - 240$

$P(15) = 0$

Hence proved that 15 is the right answer to get 15 as the zero of resulting polynomial.

26. Question

A quadratic polynomial, the sum of whose zeroes is 0 and one zero is 3, is

A. $x^2 - 9$

B. $x^2 + 9$

C. $x^2 + 3$

D. $x^2 - 3$

Answer

Given,

One of the polynomial is 3,

Sum of the zeros = 0

Now

Let the other zero be x,

$x + 3 = 0$

$x = - 3$

So,

Zeros of the polynomial are 3 and – 3

Product of the zeros = 3 × – 3 = – 9

Polynomial with zeros 3 and – 3 = $x^2 - (0)x + (-9) = x^2 - 9$

27. Question

If two zeroes of the polynomial $x^3 + x^2 - 9x - 9$ are 3 and – 3, then its third zero is

A. – 1

B. 1

C. – 9

D. 9

Answer

Given

$x^3 + x^2 - 9x - 9$

Zeros = 3 and – 3

$$\begin{array}{r|l} & x+1 \\ \hline x^2-9 & x^3+x^2-9x-9 \\ & -x^3-9x \\ & \quad + \\ \hline & x^2-9 \end{array}$$

x = – 1

So its third zero is – 1

28. Question

If √5 and – √5 are two zeroes of the polynomial $x^3 + 3x^2 - 5x - 15$, then its third zero is

A. 3

B. – 3

C. 5

D. – 5

Answer

Given,

$x^3 + 3x^2 - 5x - 15$

Zeros = √5 and – √5

$$\begin{array}{r} x+1 \\ x^2-9\overline{)x^3+x^2-9x-9} \\ -x^3-9x \\ + \\ \hline x^2-9 \end{array}$$

So the third zero = − 3

29. Question

If x + 2 is a factor of $x^2 + ax + 2b$ and a + b = 4, then

A. a = 1,b = 3

B. a = 3,b = 1

C. a = − 1,b = 5

D. a = 5,b = − 1

Answer

Given;

A factor = x + 2

So,

x + 2 = 0

x = − 2

$p(x) = x^2 + ax + 2b$

$p(-2) = (-2)^2 + (-2)a + 2b$

0 = 4 − 2a + 2b

− 4 = − 2 (a − b)

2 = a − b..............Equation (i)

4 = a + b …….. Equation (ii)

By Adding (i) and (ii) we get,

2 − a + b + 4 − a − b = − 2a + 2

2a = 2

So we get, a = 3 and b = 1

30. Question

The polynomial which when divided by $-x^2 + x - 1$ gives a quotient x − 2 and remainder 3, is

A. $x^3 - 3x^2 + 3x - 5$

B. $-x^3 - 3x^2 - 3x - 5$

C. $-x^3 + 3x^2 - 3x + 5$

D. $x^3 - 3x^2 - 3x + 5$

Answer

Given,

Divisor = $-x^2 + x - 1$

Quotient = $x - 2$

Remainder = 3

Now we have to find out Dividend…

As we know that,

Dividend = Divisor × Quotient + Remainder

Dividend = $(-x^2 + x - 1)(x - 2) + 3$

$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$

$= -x^3 + 3x^2 - 3x + 5$

So,

The required polynomial is $-x^3 + 3x^2 - 3x + 5$